CS423: Data Mining

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Eigenvector and Eigenvalue

- Definition 1: A nonzero vector x is an eigenvector (or characteristic vector) of a square matrix A if there exists a scalar λ such that $Ax = \lambda x$.
- Then λ is an eigenvalue (or characteristic value) of A.
- Note: The zero vector can not be an eigenvector even though $A0 = \lambda 0$.
- But $\lambda = 0$ can be an eigenvalue.

Geometric interpretation

- An $n \times n$ matrix A multiplied by $n \times 1$ vector x results in another $n \times 1$ vector $y = Ax$. Thus A can be considered as a transformation matrix.
- In general, a matrix acts on a vector by changing both its magnitude and its direction. However, a matrix may act on certain vectors by changing only their magnitude, and leaving their direction unchanged (or possibly reversing it). These vectors are the eigenvectors of the matrix.

Example

- Show $x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is an eigenvector for $A = \begin{bmatrix} 2 & -4 \\ 3 & -6 \end{bmatrix}$
- Solution: $Ax = \ldots$

Finding Eigenvectors

- Given A $= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, find its eigenvectors and eigenvalues
- Solution:
    1. from definition $Ax = \lambda x$
    2. $(A - \lambda I)x = 0$
    3. Since x is nonzero, we know that $(A - \lambda I)$ is not invertible.
    4. So determinant of $(A - \lambda I)$ must be zero.
    5. $|A - \lambda I| = 0$

Properties of Eigenvectors

- All eigenvalues of a real symmetric matrix are real
- For symmetric matrices, eigenvectors for distinct eigenvalues are orthogonal
- An m-by-m square matrix can have at most m distinct eigenvectors