Algorithms

Master method

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สมการที่จะใช้ master method ได้ต้องอยู่ในรูป

$$T(n) = aT(n/b) + f(n)$$

โดยที่

- $a \geq 1$, $b > 1$ เป็นค่าคงที่
- $f(n)$ เป็นค่าบวก
- แยกเป็น 3 กรณี

Let $a \geq 1$ and $b > 1$ be constants, let $f(n)$ be a function,

Let $T(n)$ be defined on nonnegative integers by the recurrence

$T(n) = aT(n/b) + f(n)$, where we can replace n/b by $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$.

$T(n)$ can be bounded asymptotically in three cases:

1. If $f(n) = O(n^{\log_b a - \varepsilon})$ for some constant $\varepsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$.

2. If $f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \log n)$.

3. If $f(n) = \Omega(n^{\log_b a + \varepsilon})$ for some constant $\varepsilon > 0$,

   and if, for some constant $c < 1$ and all sufficiently large n,

   we have $a \cdot f(n/b) \leq c\, f(n)$, then $T(n) = \Theta(f(n))$.

$T(n) = 16T(n/4)+n$

$a = 16, b = 4, f(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^2.$

ทดสอบ $f(n) = n = O(n^{\log_b a - \varepsilon}) = O(n^{2-\varepsilon})$, $\varepsilon > 0$?

Yes, เมื่อ $\varepsilon = 1$ ตรงกับ Case 1.

ดังนั้น, $T(n) = \Theta(n^{\log_b a}) = \Theta(n^2)$.

2. $T(n) = T(3n/7) + 1$

$a = 1, b=7/3, f(n) = 1$

$n^{\log_b a} = n^{\log_{(7/3)} 1} = n^0 = 1$

ทดสอบ $f(n) = 1 = O(n^{\log_b a - \varepsilon}) = O(n^{0-\varepsilon})$, $\varepsilon > 0$? No

ต่อไปทดสอบ, Is $f(n) = 1 = \Theta(n^{\log_b a})$? ทำได้ Yes ตรงกับ Case 2.

ดังนั้น, $T(n) = \Theta(n^{\log_b a} \log n) = \Theta(\log n)$

3. $T(n) = 3T(n/4) + n \log n$

$a = 3$, $b=4$, thus $n^{\log_b a} = n\log 43 = O(n^{0.793})$

Is $f(n) = n \log n = O(n^{\log_4 3 - \varepsilon})$, $\varepsilon > 0$? No

but, $f(n) = n \log n = \Omega(n^{\log_4 3 + \varepsilon})$ where $\varepsilon \approx 0.2$ ตรงกับ Case 3.

and for sufficiently large n,

$a\, f(n/b) = 3(n/4 \log n/4) \leq c\,(n \log n)$

$= cf(n)$ is true where $c = 3/4 < 1$

Therefore, $T(n) = \Theta(f(n)) = \Theta(n \log n)$.

$T(n)=T(n/2)+O(1)$, $T(1)=1$

4. $T(n) = 2T(n/2) + n \log n$

$a = 2$, $b=2$, $f(n) = n \log n$, and $n^{\log_b a} = n^{\log_2 2} = n$

$f(n)$ is asymptotically larger than $n^{\log_b a}$, but not polynomially larger. The ratio $\log n$ is asymptotically less than n^{ε} for any positive ε. Thus, the Master Theorem doesn't apply here.