

Introduction to Data Science



Chapter 4

Predictive Analysis

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Outline

Predictive Analysis

1. Predictive Analysis

- Preparing Datasets

2. Classification Analysis

- K-Nearest Neighbor
- Decision Tree
- Naïve Bayes
- Classification Assessment

3. Regression Analysis

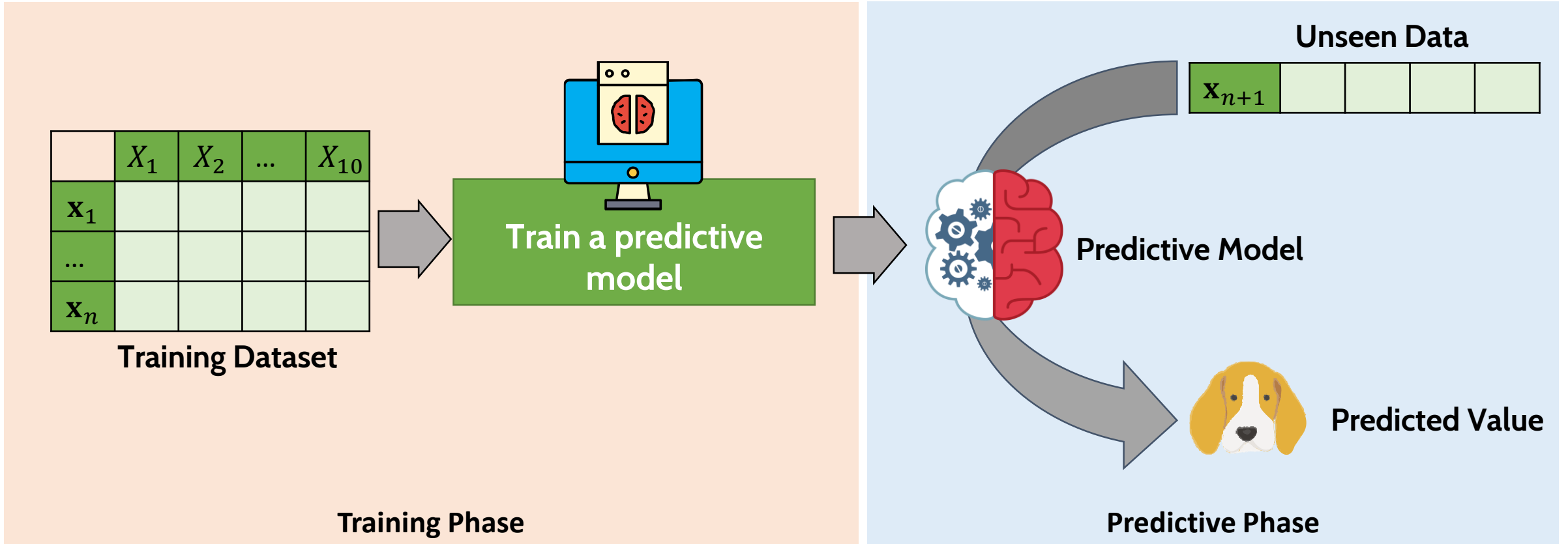
- Linear Regression
- Polynomial Regression
- Regression Assessment

4. Time Series Analysis

- Moving Average
- Autoregressive Integrated Moving
- Curve Fitting

Predictive Analysis

Analyze current and historical data to make predictions about future or otherwise unknown events.



Preparing Dataset

Predictive Analysis

	Features				Target values
D	X_1	X_2	...	X_{10}	Y
x_1					
x_2					
x_3					
...					
x_l					
x_{l+1}					
x_{l+2}					
...					
x_n					

To perform a predictive analysis:

- We should have two datasets: training and test datasets.
- The target value of each datapoint must be available.

Training dataset

- Will be used to train a predictive model.
- Target value of each data point must be available.

Test dataset

- Will be used to evaluate the predictive model
- Assume that target value of each data point is not known, but it should be available.

Classification Analysis

	Features				Target class
D	X_1	X_2	...	X_{10}	Y
x_1					
x_2					
x_3					
...					
x_l					
x_{l+1}					
x_{l+2}					
...					
x_n					

For classification analysis

- The value we want to predict is **categorical data**.
- Known as **class**

Example

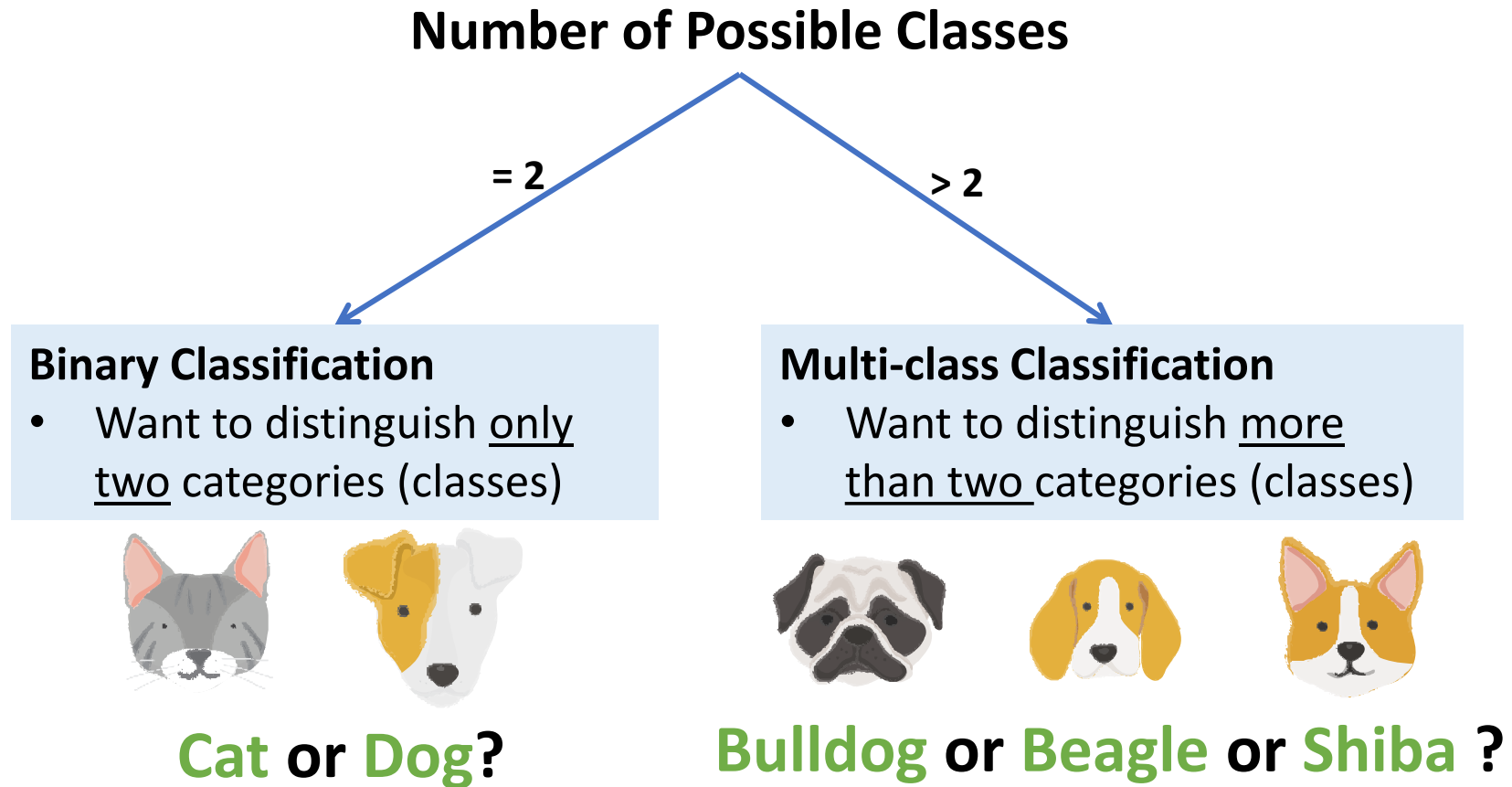
We know some characteristics of an animal, and we want to predict it is a cat or a dog.



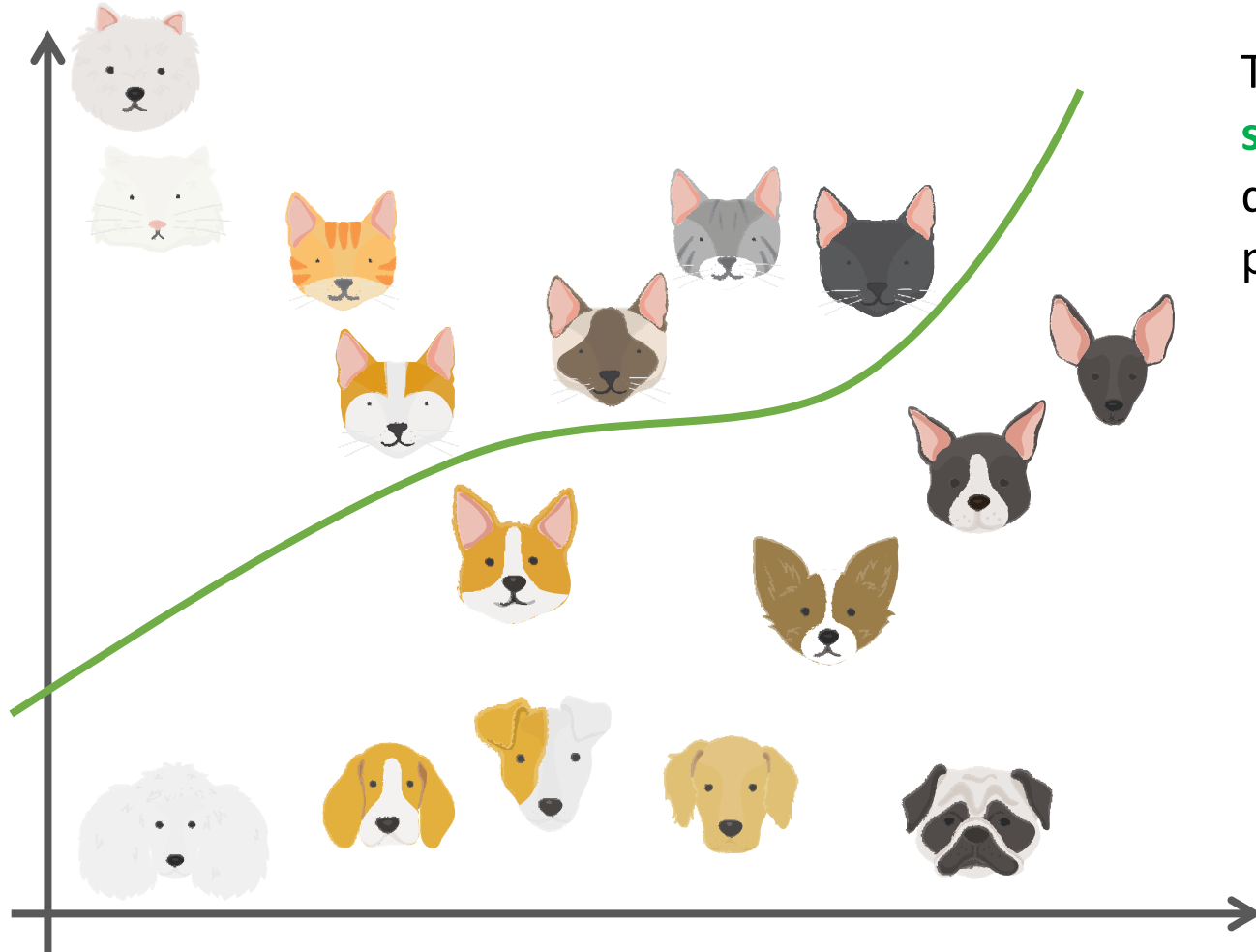
cat or dog?

Classification Analysis

Types of Classification Problems



Classification Analysis



The task of classification is one of finding **separating lines** that separate classes of data from a training dataset as best as possible.

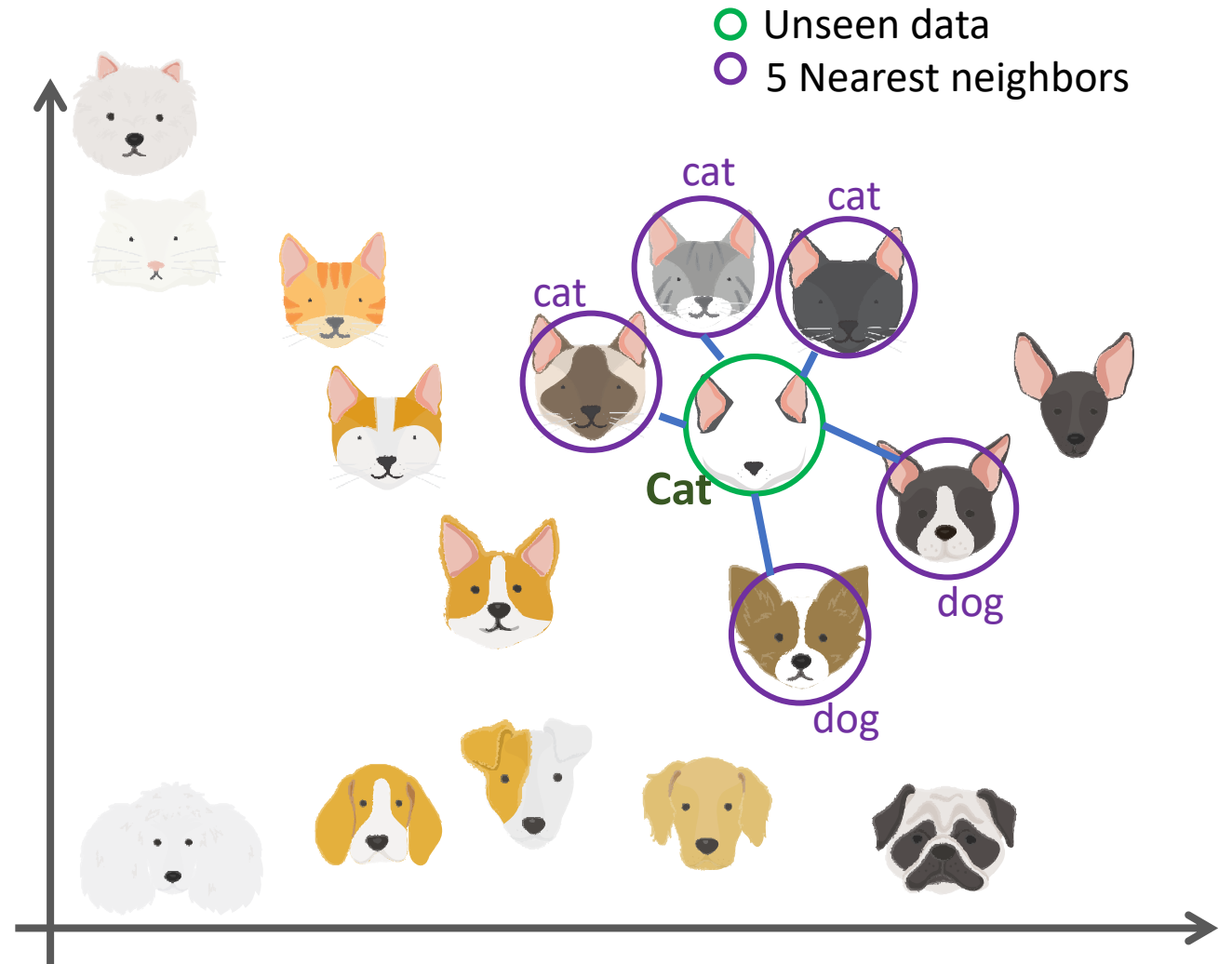
K-Nearest Neighbor

Classification Analysis

K-Nearest Neighbor classifier assigns the class label of an unseen data with the majority class labels of k neighbor data (in the training dataset)

How the k-nearest neighbor works

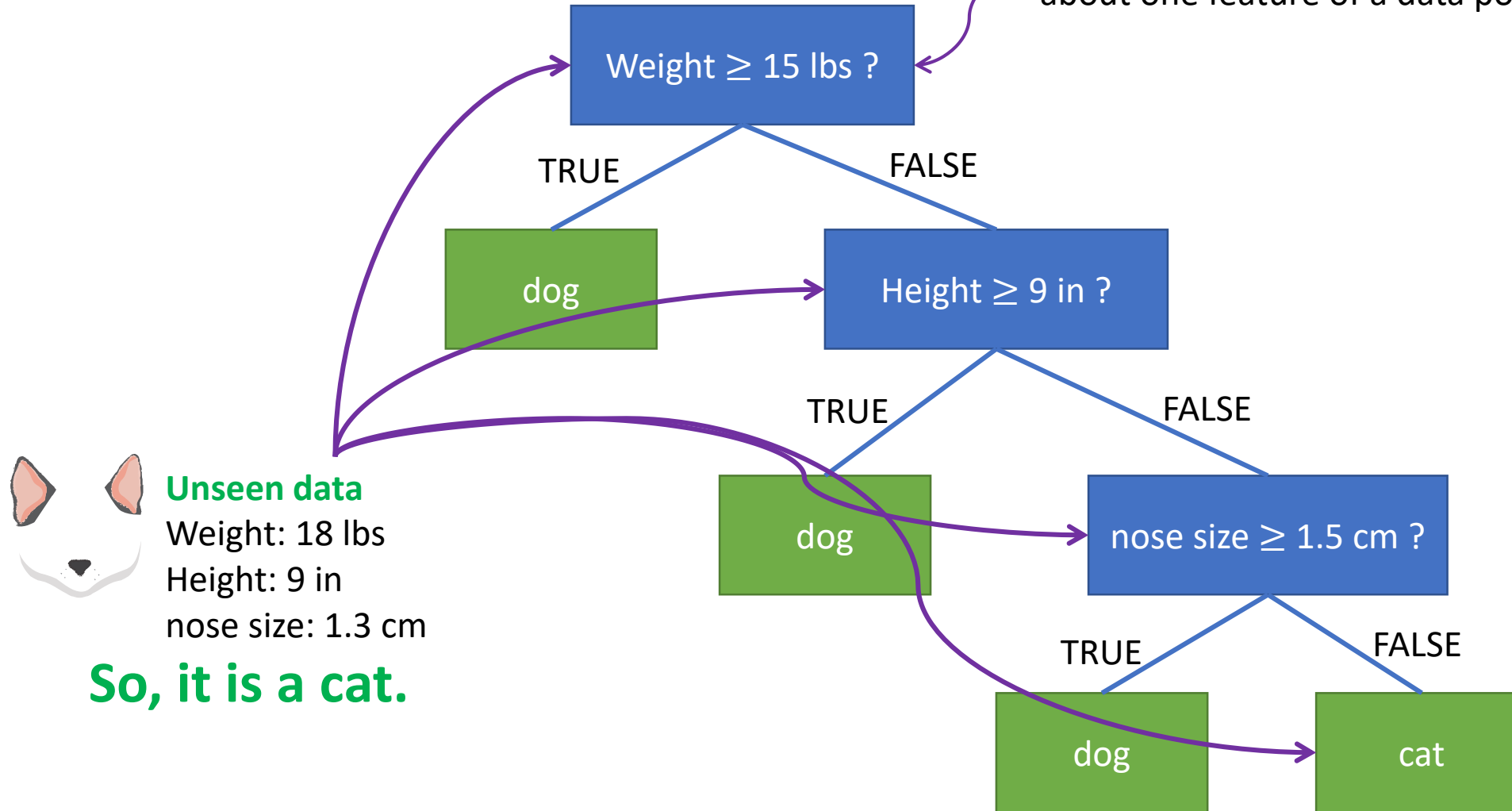
- STEP 1: Calculate distances between an unseen data and training data
- STEP 2: Find k nearest neighbor
- STEP 3: Find majority class label
- STEP 4: Assign the majority class label to the class label of the unseen data



Decision Tree

Classification Analysis

Every node in the tree asks a question about one feature of a data point.



Decision Tree

Classification Analysis

Construct a decision tree

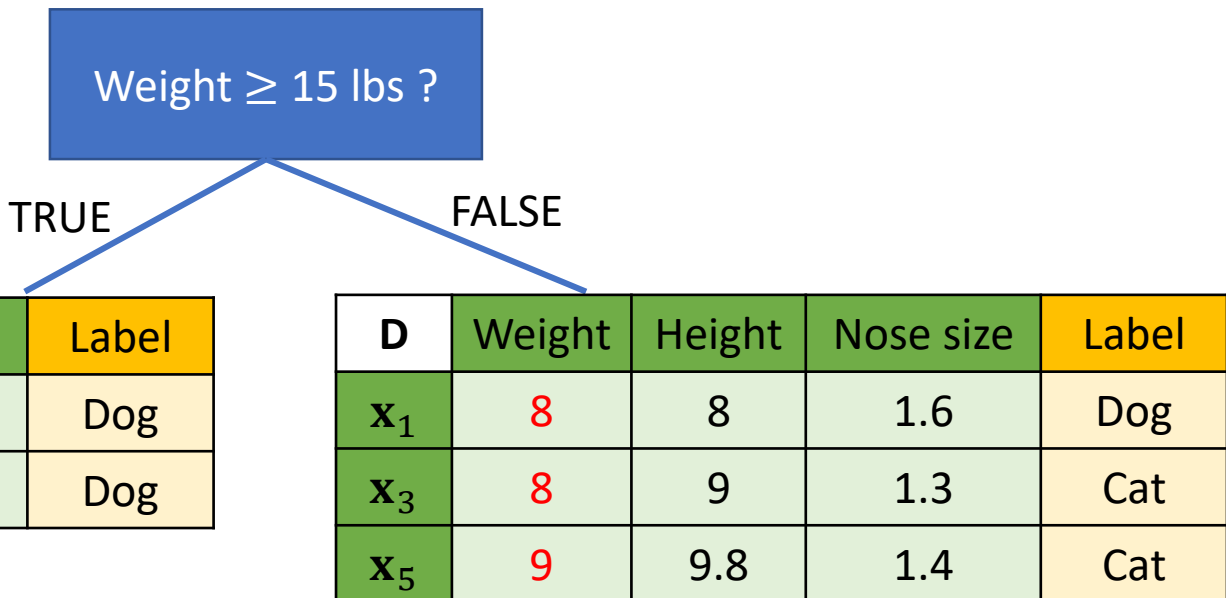
STEP 1: Given a training data D , find the single feature (and cutoff for that feature, if it's numerical) that best partitions your data into classes.

STEP 2: This single best feature/cutoff becomes the root of your decision tree.

STEP 3: Partition D up according to the root node.

STEP 4: Recursively train each of the child nodes on its partition of the data until all of the data points in the partition have the same label.

D	Weight	Height	Nose size	Label
x_1	8	8	1.6	Dog
x_2	50	40	3	Dog
x_3	8	9	1.3	Cat
x_4	15	12	2.5	Dog
x_5	9	9.8	1.4	Cat

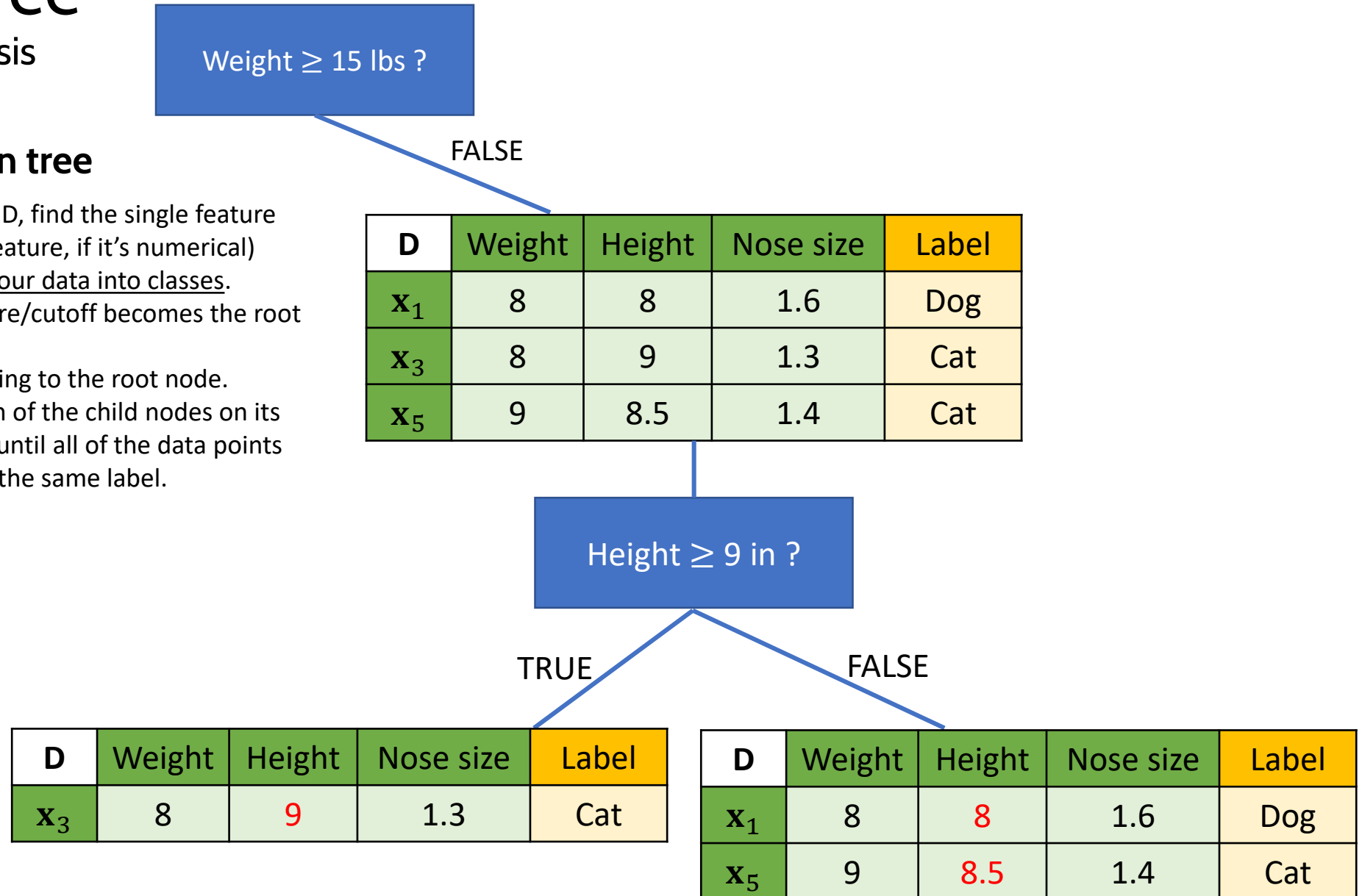


Decision Tree

Classification Analysis

Construct a decision tree

- STEP 1: Given a training data D , find the single feature (and cutoff for that feature, if it's numerical) that best partitions your data into classes.
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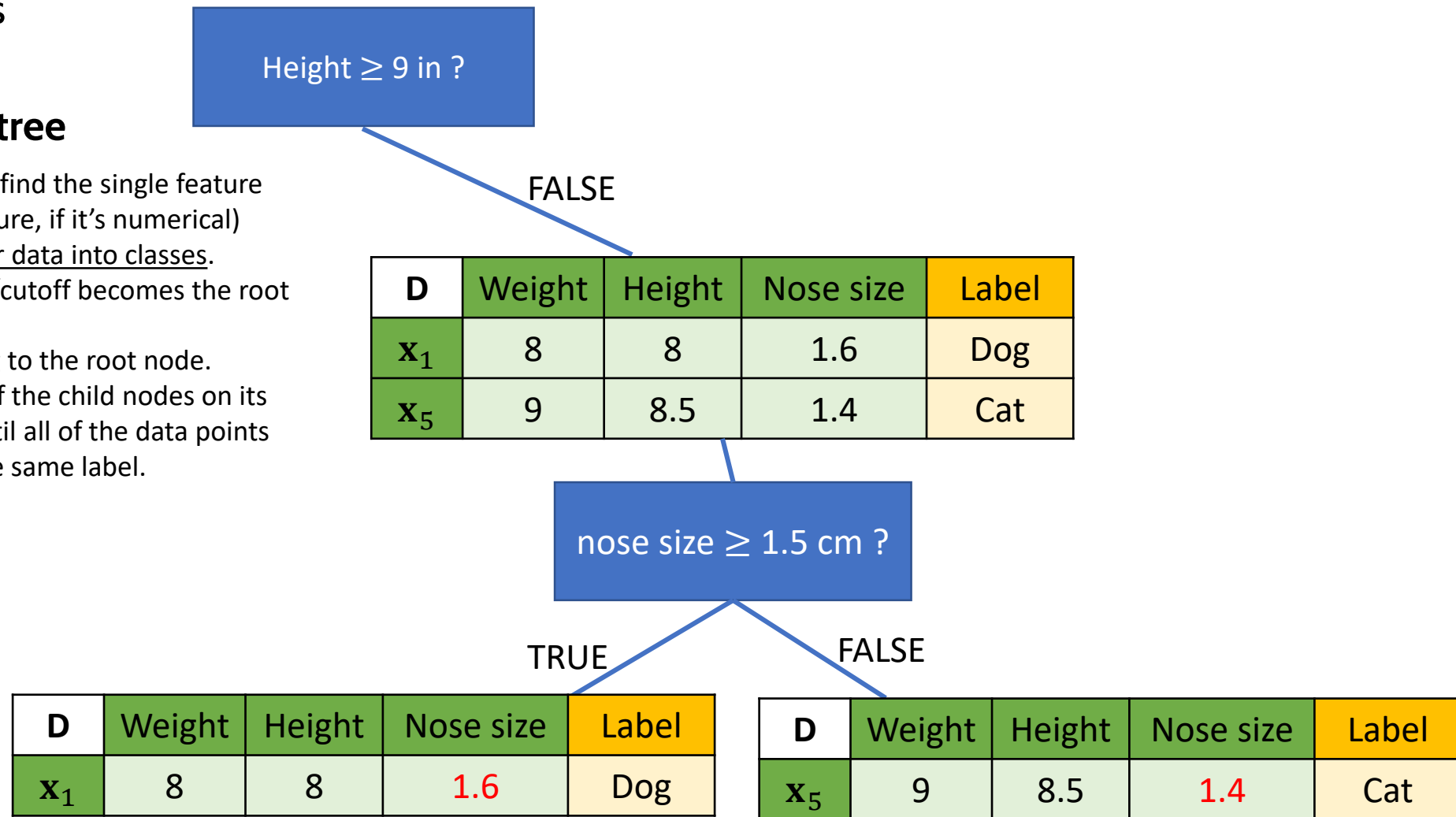


Decision Tree

Classification Analysis

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Decision Tree

Classification Analysis

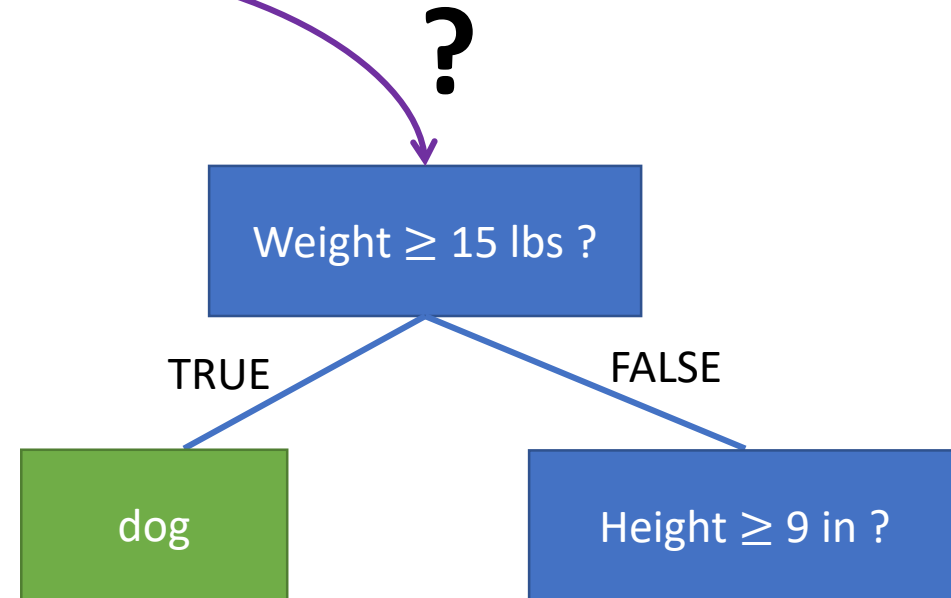
How to determine the best feature and cutoff

The most common ones are:

- Information gain
- Gini impurity.

You can find more details in:

- Zaki, M., & Meira, W. (2014). Data mining and analysis : Fundamental concepts and algorithms. New York: Cambridge University Press.
- https://en.wikipedia.org/wiki/Decision_tree_learning



Naïve Bayes

Classification Analysis

Bayes Theorem:

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)}$$

Probability of **A** happening,
given that **B** has occurred

The *prior*, the initial
degree of belief in **A**.

The likelihood of event **B**
occurring given that **A** is
true.



Thomas Bayes
1701-1761

Source:

https://en.wikipedia.org/wiki/Thomas_Bayes#/media/File:Thomas_Bayes.gif

Naïve Bayes

Classification Analysis

Classify whether the day is suitable for playing golf, given the features of the day.

Bayes theorem can be rewritten as:

$$P(y|\mathbf{x}) = \frac{P(y)P(\mathbf{x}|y)}{P(\mathbf{x})}$$

We want to classify

$\mathbf{x} = (\text{Sunny, Hot, Normal, True})$

D	Outlook	Temperature	Humidity	Windy	Play golf
\mathbf{x}_1	Rainy	Hot	High	False	No
\mathbf{x}_2	Rainy	Hot	High	True	No
\mathbf{x}_3	Overcast	Hot	High	False	Yes
\mathbf{x}_4	Sunny	Mild	High	False	Yes
\mathbf{x}_5	Sunny	Cool	Normal	False	Yes
\mathbf{x}_6	Sunny	Cool	Normal	True	No
\mathbf{x}_7	Overcast	Cool	Normal	True	Yes
\mathbf{x}_8	Rainy	Mild	High	False	No
\mathbf{x}_9	Rainy	Cool	Normal	False	Yes
\mathbf{x}_{10}	Sunny	Mild	Normal	False	Yes
\mathbf{x}_{11}	Rainy	Mild	Normal	True	Yes
\mathbf{x}_{12}	Overcast	Mild	High	Ture	Yes
\mathbf{x}_{13}	Overcast	Hot	Normal	False	Yes
\mathbf{x}_{14}	Sunny	Mild	High	True	No

Naïve Bayes

Classification Analysis

How the Naïve Bayes works

STEP 1: Calculate $P(y)$ for all possible value of y from the training dataset.

STEP 2: Calculate $P(\mathbf{x}|y) = \prod_{i=1}^p P(x_i|y)$ for all possible value of y from the training dataset.

STEP 3: Calculate $P(y|\mathbf{x}) = P(y) \prod_{i=1}^p P(x_i|y)$

STEP 4: Assign y that reach the highest $P(y|\mathbf{x})$ to the class label of \mathbf{x}

We want to classify

$\mathbf{x} = (\text{Sunny, Hot, Normal, True})$

D	Outlook	Temperature	Humidity	Windy	Play golf
\mathbf{x}_1	Rainy	Hot	High	False	No
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STEP 4: Assign y that reach the highest $P(y|\mathbf{x})$ to the class label of \mathbf{x}

$$P(\text{Play golf} = \text{No}) = \frac{5}{14}$$

$$P(\text{Play golf} = \text{Yes}) = \frac{9}{14}$$

We want to classify

$\mathbf{x} = (\text{Sunny, Hot, Normal, True})$

D	Outlook	Temperature	Humidity	Windy	Play golf
\mathbf{x}_1	Rainy	Hot	High	False	No
\mathbf{x}_2	Rainy	Hot	High	True	No
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$$P(\text{Outlook} = \text{Sunny} | \text{Play golf} = \text{No}) = \frac{2}{5}$$

$$P(\text{Outlook} = \text{Sunny} | \text{Play golf} = \text{Yes}) = \frac{3}{9}$$

We want to classify

$\mathbf{x} = (\text{Sunny}, \text{Hot}, \text{Normal}, \text{True})$

D	Outlook	Temperature	Humidity	Windy	Play golf
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$$P(\text{Temperature} = \text{Hot} | \text{Play golf} = \text{Yes}) = \frac{2}{9}$$

We want to classify

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STEP 4: Assign y that reach the highest $P(y|\mathbf{x})$ to the class label of \mathbf{x}

$$P(\text{Humidity} = \text{Normal} | \text{Play golf} = \text{No}) = \frac{1}{5}$$

$$P(\text{Humidity} = \text{Normal} | \text{Play golf} = \text{Yes}) = \frac{6}{9}$$

We want to classify

$\mathbf{x} = (\text{Sunny, Hot, Normal, True})$

D	Outlook	Temperature	Humidity	Windy	Play golf
\mathbf{x}_1	Rainy	Hot	High	False	No
\mathbf{x}_2	Rainy	Hot	High	True	No
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Naïve Bayes

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$$P(\text{Windy} = \text{True} | \text{Play golf} = \text{No}) = \frac{3}{5}$$

$$P(\text{Windy} = \text{True} | \text{Play golf} = \text{Yes}) = \frac{3}{9}$$

We want to classify

$\mathbf{x} = (\text{Sunny}, \text{Hot}, \text{Normal}, \text{True})$

D	Outlook	Temperature	Humidity	Windy	Play golf
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Naïve Bayes

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$$P(\text{Play golf} = \text{No} | \text{Sunny, Hot, Normal, True}) \\ = \frac{5}{14} \times \frac{2}{5} \times \frac{2}{5} \times \frac{1}{5} \times \frac{3}{5} = 0.0069$$

$$P(\text{Play golf} = \text{Yes} | \text{Sunny, Hot, Normal, True}) \\ = \frac{9}{14} \times \frac{3}{9} \times \frac{2}{9} \times \frac{6}{9} \times \frac{3}{9} = \mathbf{0.0106}$$

So, it is suitable to **play golf** given the conditions (Outlook = Sunny, Temperature = Hot, Humidity = Normal and Windy = True).

We want to classify

$\mathbf{x} = (\text{Sunny, Hot, Normal, True})$

$$P(\text{Play golf} = \text{No}) = \frac{5}{14}$$

$$P(\text{Play golf} = \text{Yes}) = \frac{9}{14}$$

$$P(\text{Outlook} = \text{Sunny} | \text{Play golf} = \text{No}) = \frac{2}{5}$$

$$P(\text{Outlook} = \text{Sunny} | \text{Play golf} = \text{Yes}) = \frac{3}{9}$$

$$P(\text{Temperature} = \text{Hot} | \text{Play golf} = \text{No}) = \frac{2}{5}$$

$$P(\text{Temperature} = \text{Hot} | \text{Play golf} = \text{Yes}) = \frac{2}{9}$$

$$P(\text{Humidity} = \text{Normal} | \text{Play golf} = \text{No}) = \frac{1}{5}$$

$$P(\text{Humidity} = \text{Normal} | \text{Play golf} = \text{Yes}) = \frac{6}{9}$$

$$P(\text{Windy} = \text{True} | \text{Play golf} = \text{No}) = \frac{3}{5}$$

$$P(\text{Windy} = \text{True} | \text{Play golf} = \text{Yes}) = \frac{3}{9}$$

Naïve Bayes

Classification Analysis

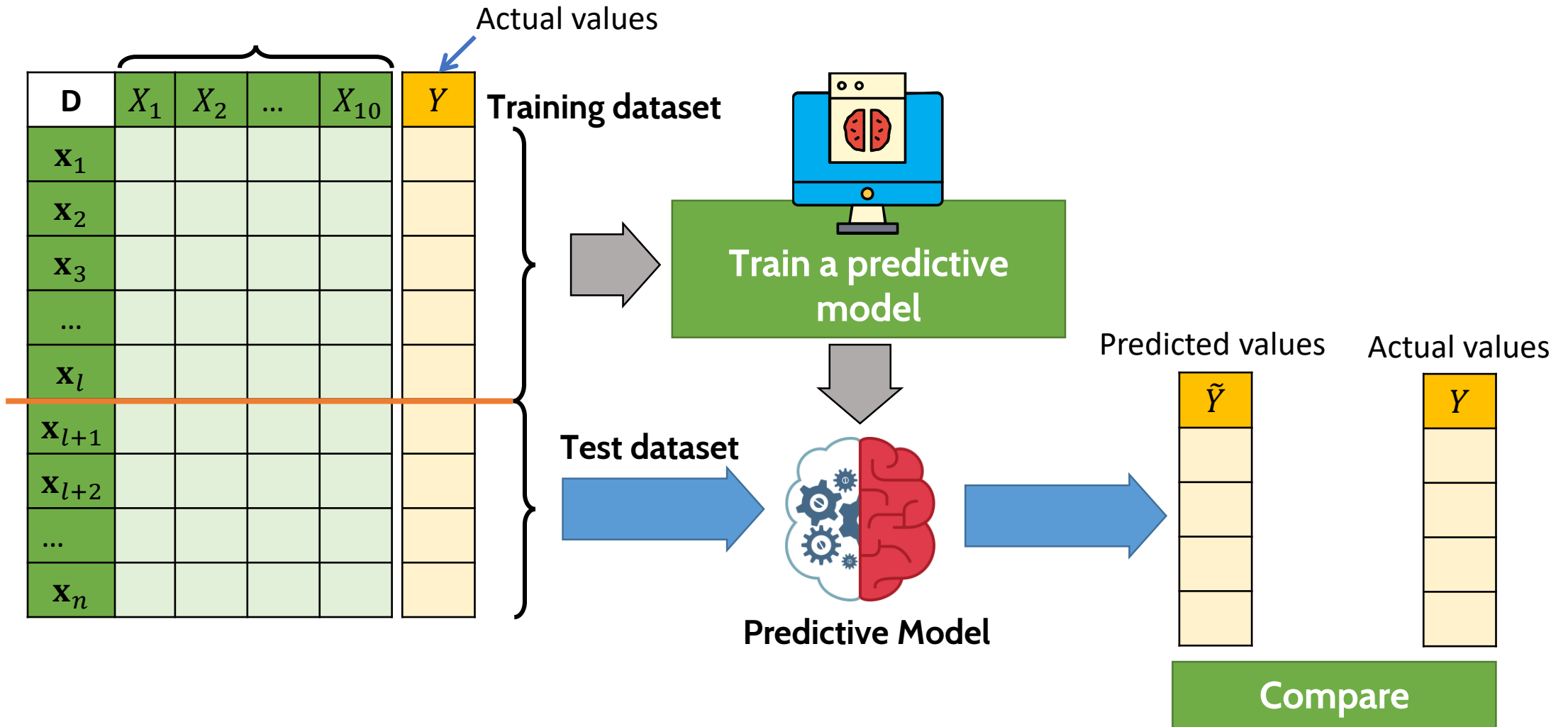
Quiz:

It is suitable to play golf or not given the conditions (Outlook = Rainy, Temperature = Mild, Humidity = Normal and Windy = False).

D	Outlook	Temperature	Humidity	Windy	Play golf
x_1	Rainy	Hot	High	False	No
x_2	Rainy	Hot	High	True	No
x_3	Overcast	Hot	High	False	Yes
x_4	Sunny	Mild	High	False	Yes
x_5	Sunny	Cool	Normal	False	Yes
x_6	Sunny	Cool	Normal	True	No
x_7	Overcast	Cool	Normal	True	Yes
x_8	Rainy	Mild	High	False	No
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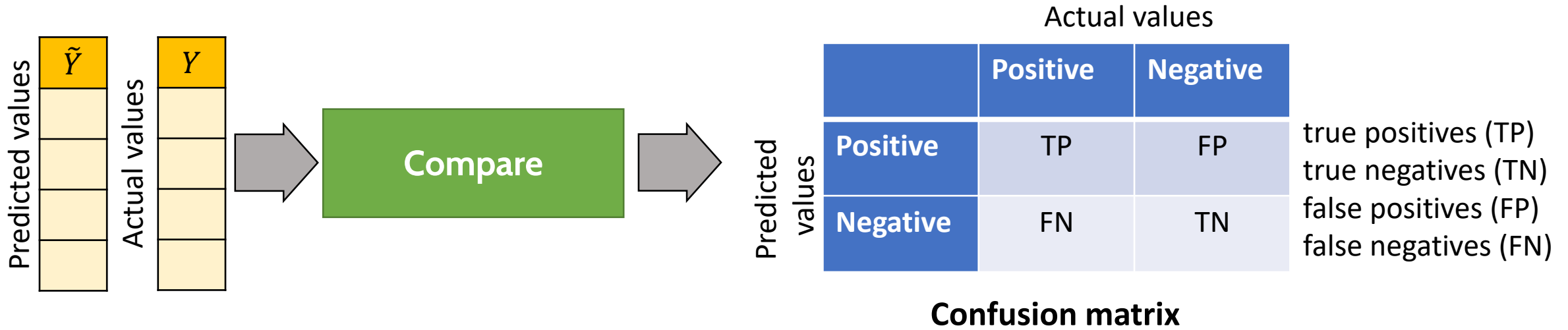
Classification Assessment

Classification Analysis



Classification Assessment

Classification Analysis



$$\text{Accuracy} = \frac{(\text{TP} + \text{TN})}{\text{Total}}$$

$$\text{Misclassification Rate} = \frac{(\text{FP} + \text{FN})}{\text{Total}} = 1 - \text{Accuracy}$$

$$\text{Recall} = \frac{\text{TP}}{\text{TP} + \text{FN}}$$

$$\text{Precision} = \frac{\text{TP}}{\text{TP} + \text{FP}}$$

Classification Assessment

Classification Analysis

Example

		Actual values		
		setosa	versicolor	virginica
Predicted values	setosa	10	2	4
	versicolor	1	16	1
	virginica	0	2	9

$$\text{Recall}_{\text{virginica}} = ?$$

$$\text{Precision}_{\text{virginica}} = ?$$

$$\text{Accuracy} = \frac{(10 + 16 + 9)}{45} = \frac{35}{45} = 0.78$$

$$\text{Misclassification Rate} = 1 - 0.78 = 0.22$$

$$\text{Recall}_{\text{setosa}} = \frac{10}{10 + 1 + 0} = \frac{10}{11} = 0.91$$

$$\text{Precision}_{\text{setosa}} = \frac{10}{10 + 2 + 4} = \frac{10}{16} = 0.625$$

$$\text{Recall}_{\text{versicolor}} = \frac{16}{2 + 16 + 2} = \frac{16}{20} = 0.8$$

$$\text{Precision}_{\text{versicolor}} = \frac{16}{1 + 16 + 1} = \frac{16}{18} = 0.89$$

Classification Assessment

Classification Analysis

Example

		Actual values	
		Cat	Dog
Predicted values	Cat	5	2
	Dog	3	3

$$\text{Accuracy} = \frac{(5 + 3)}{13} = \frac{8}{13} = 0.62$$

$$\text{Misclassification Rate} = \frac{(2 + 3)}{13} = \frac{5}{13} = 0.38$$

$$\text{Recall} = \frac{5}{5 + 3} = \frac{5}{8} = 0.625$$

$$\text{Precision} = \frac{5}{5 + 2} = \frac{5}{7} = 0.714$$

Regression analysis

Independent variable

Dependent variable

D	X_1	X_2	...	X_{10}	Y
x_1					
x_2					
x_3					
...					
x_l					
x_{l+1}					
x_{l+2}					
...					
x_n					

For regression analysis

- The value we want to predict is **numeric data**.
- Known as **Dependent variable**

Example

- We know quantities of water and fertilizer providing to a tree for a month
- We want to predict the growth rate (height) of the tree.



Further Study

- **Book:**

- Zaki, M., & Meira, W. (2014). Data mining and analysis : Fundamental concepts and algorithms. New York: Cambridge University Press.
- Enders, C. (2010), Applied Missing Data Analysis. New York: Guilford Press.

- **Online lesson:** วิทยาการข้อมูลเบื้องต้น (Introduction to Data Science) – CMU MOOC

https://thaimooc.org/courses/course-v1:CMU-MOOC+cmu034+2019_T1/about