Introduction to Data Science

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Chapter 5
Prescriptive Analysis

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Outline

Predictive Analysis

1. **Prescriptive Analysis**
2. **Optimization**
    - **Optimization Problem**
    - **Defining an optimization problem**
    - **Applying predictive analytics to solve optimization problem**
3. **Simulation**
    - **What is simulation?**
    - **Applying Simulation to Prescriptive Analysis**
    - **Monte Carlo simulation**

Simulation

Prescriptive Analysis

The process of modelling a real-life or hypothetical situation on a computer.

Simulation

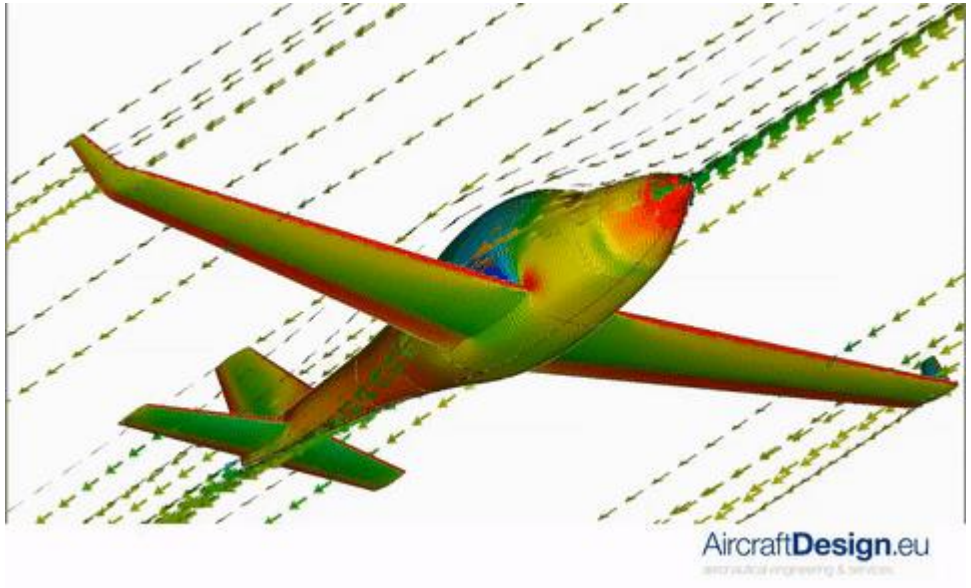
Prescriptive Analysis

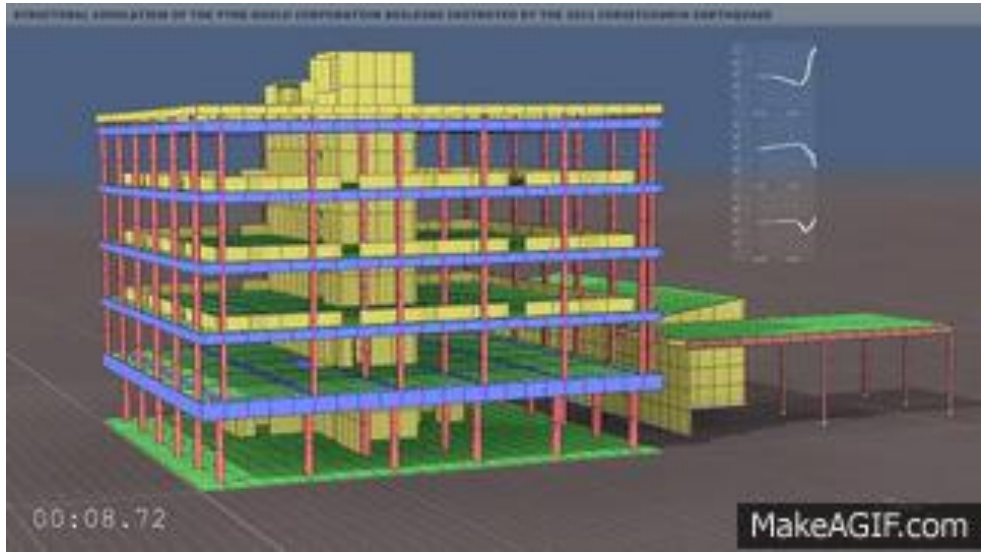
Simulation is useful because it can be used to test new ideas about business decisions and actions to mitigate risk pertaining to a process or system, or how modifications will affect an existing process or system.

Simulation

Prescriptive Analysis

Simulation is often used in applications related to safety of infrastructure as well as safety, quality, and design of products.





Simulation

Prescriptive Analysis

Applying Simulation to Prescriptive Analysis

Example: Optimum number of open Check-out counters

The aim is to experiment with different queuing systems to understand their customers better.

- For instance, there are 20 counters in total.
- Each open counter costs $20 to operate.
- Customers who wait more than 10 minutes are ‘annoyed’ and there’s a penalty of $1 per customer.

What is the optimal number of counters?

Simulation

Prescriptive Analysis

Applying Simulation to Prescriptive Analysis

Example: Optimum number of open Check-out counters

What is the optimal number of counters?

Option 1: Perform experiment of different numbers of counters on real situation.

Option 2: Simulate experiment of different numbers of counters on computer.

Simulation

Prescriptive Analysis

Applying Simulation to Prescriptive Analysis

Example: Optimum number of open Check-out counters

Assumption:

Each counter has its queue and arriving customer joins one queue randomly.

Random under a certain distribution

Gamma distributions are assumed for inter-arrival time and service time for customers.

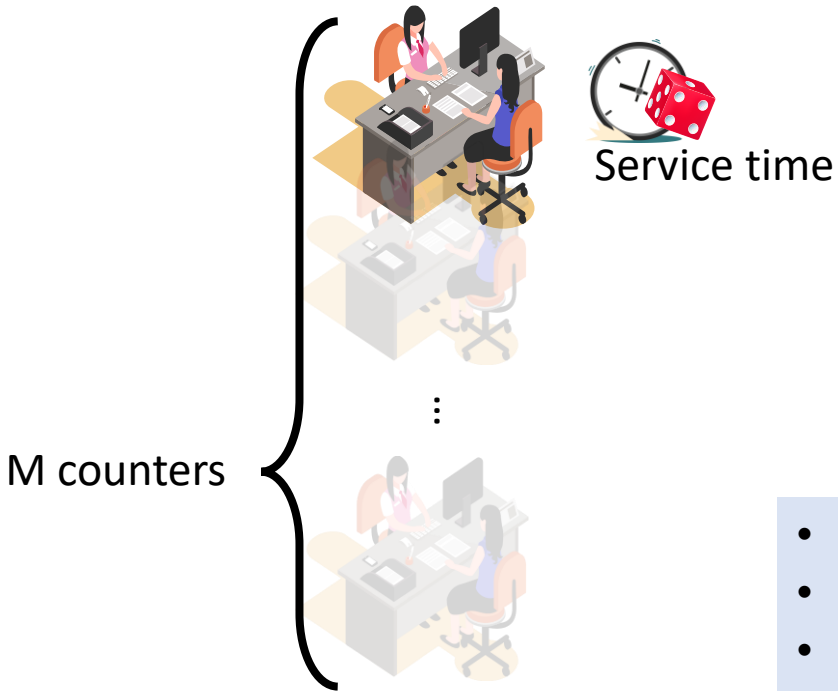
Simulation

Prescriptive Analysis

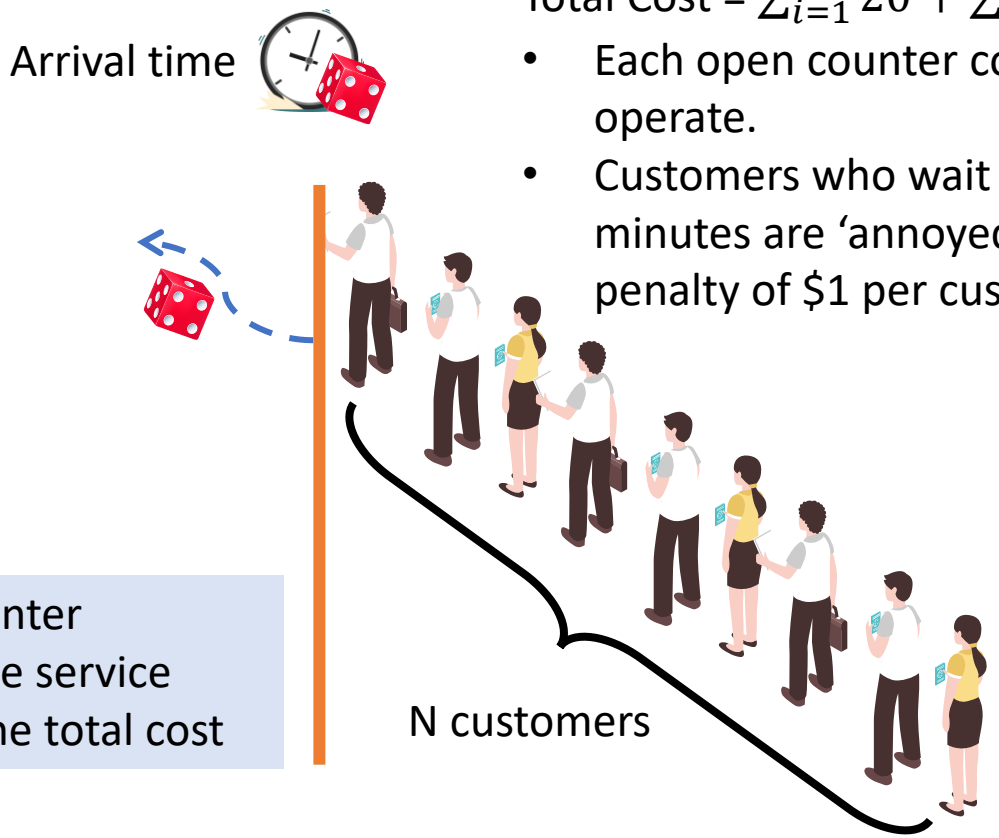
Applying Simulation to Prescriptive Analysis

Example: Optimum number of open Check-out counters

Queue Simulation



- Open 1 counter
- Simulate the service
- Calculate the total cost



Total Cost = $\sum_{i=1}^{M} 20 + \sum_{j=1}^{N} (t_j^{wait} > 10)$

- Each open counter costs $20 to operate.
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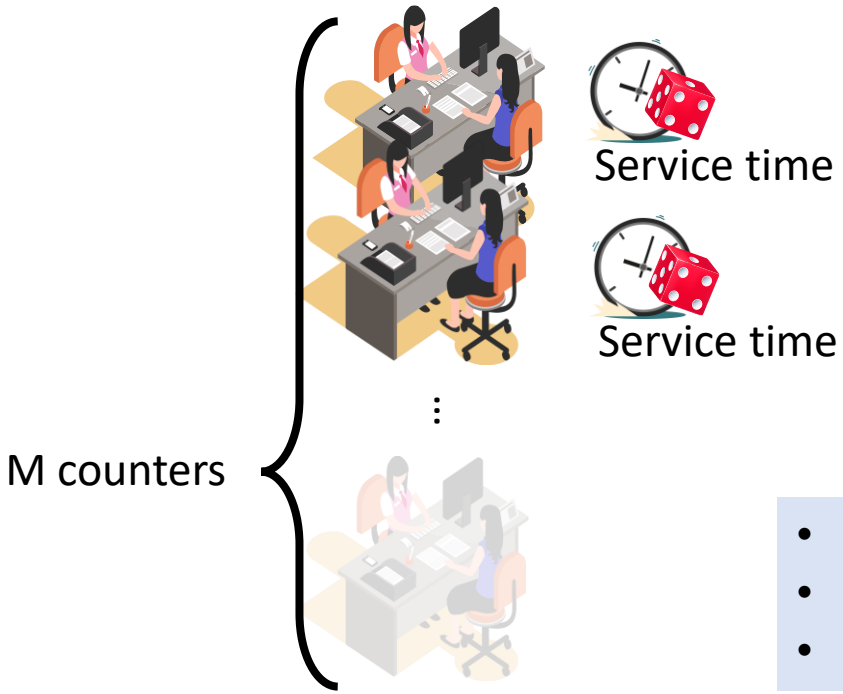
Simulation

Prescriptive Analysis

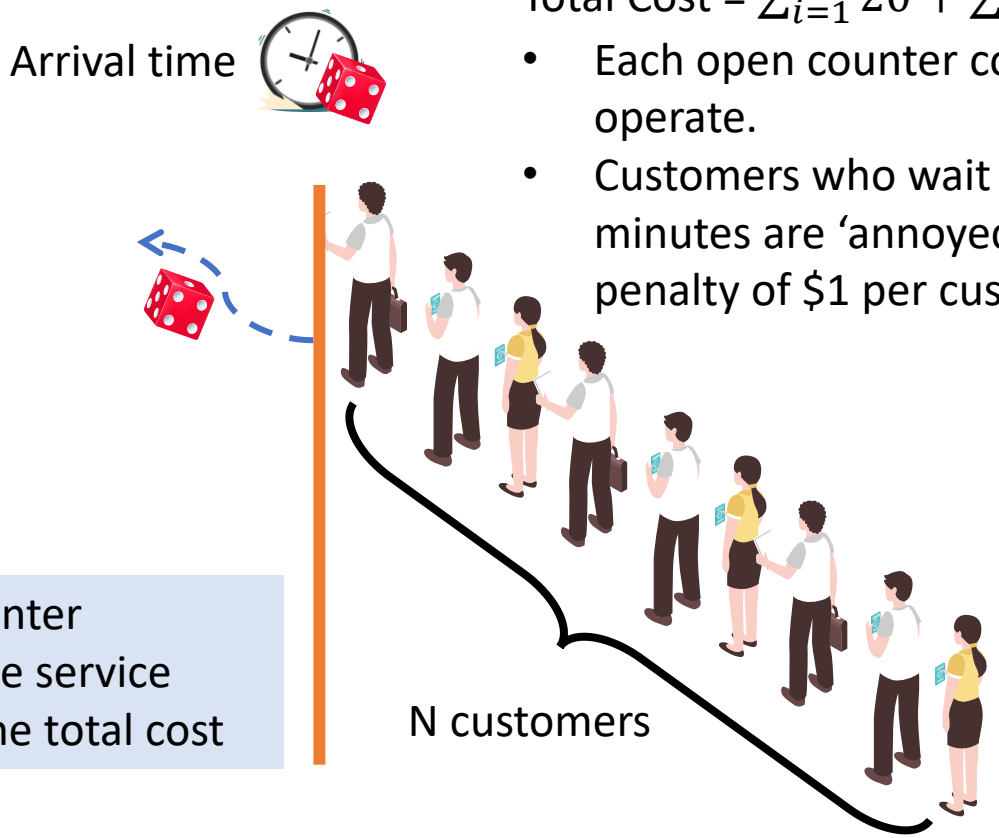
Applying Simulation to Prescriptive Analysis

Example: Optimum number of open Check-out counters

Queue Simulation



- Open 2 counter
- Simulate the service
- Calculate the total cost



Total Cost = $\sum_{i=1}^{M} 20 + \sum_{j=1}^{N} (t_j^{wait} > 10)$

- Each open counter costs $20 to operate.
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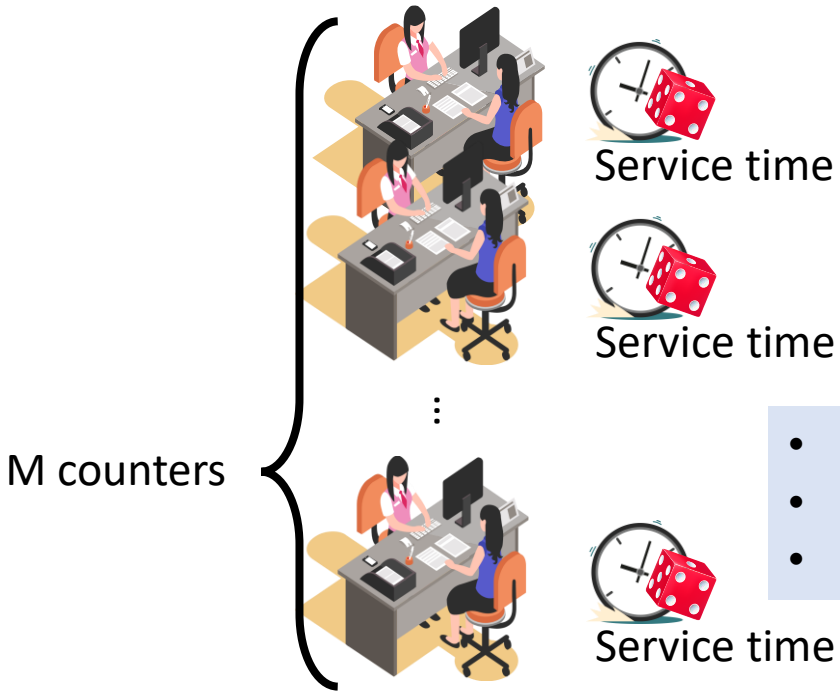
Simulation

Prescriptive Analysis

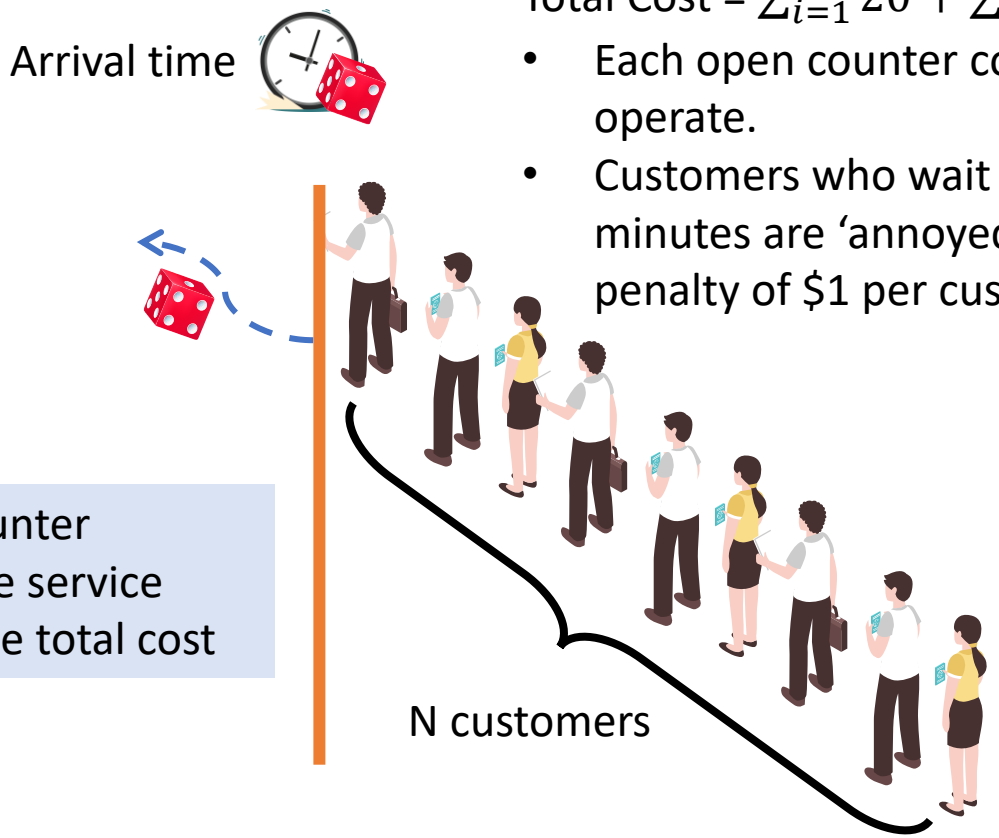
Applying Simulation to Prescriptive Analysis

Example: Optimum number of open Check-out counters

Queue Simulation



- Open M counter
- Simulate the service
- Calculate the total cost



Total Cost = $\sum_{i=1}^{M} 20 + \sum_{j=1}^{N} (t_j^{wait} > 10)$

- Each open counter costs $20 to operate.
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Simulation

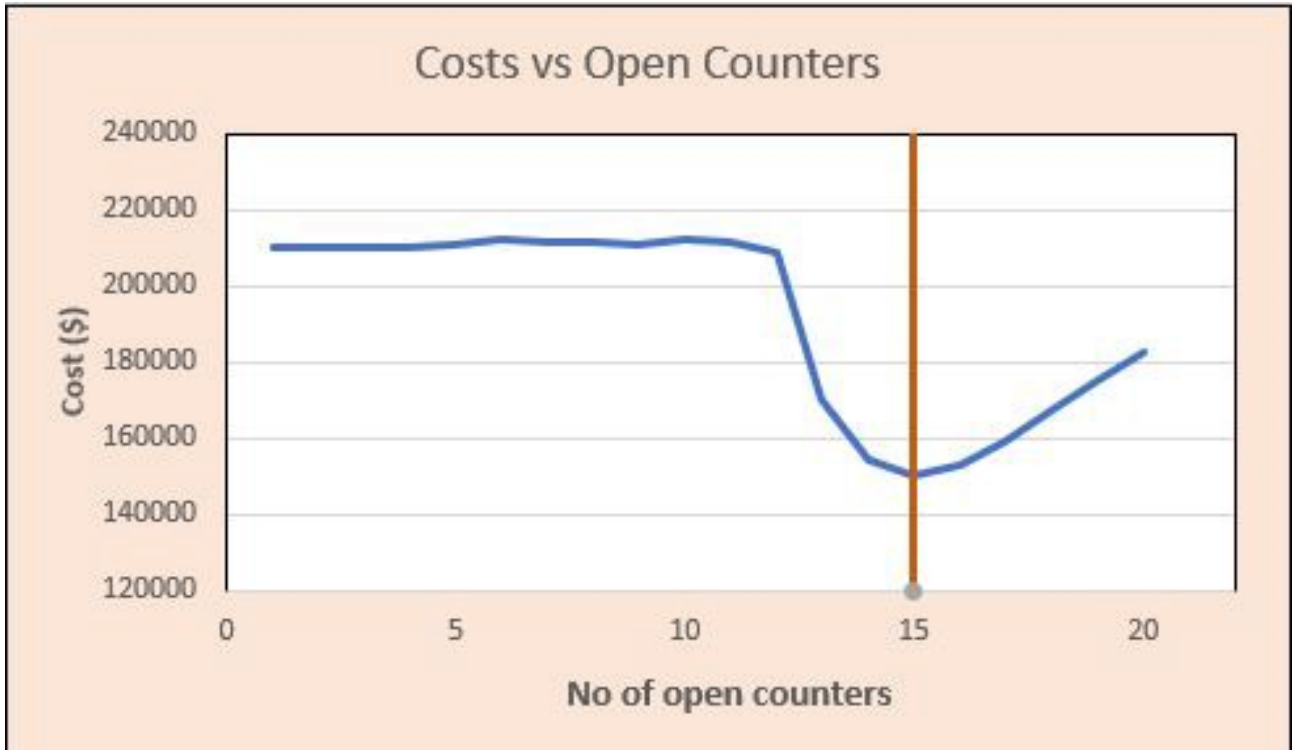
Prescriptive Analysis

Applying Simulation to Prescriptive Analysis

Example: Optimum number of open Check-out counters

Queue Simulation

15 counters achieve the minimum cost



Source: https://towardsdatascience.com/every-data-scientist-needs-to-read-these-simulation-stories-7be0531e782f

Simulation

Prescriptive Analysis

As can be seen,

Random Generator is invoked in simulation systems.

For example:

- Arrival time of each customer
- Service time for a customer

} Random under Gamma distributions

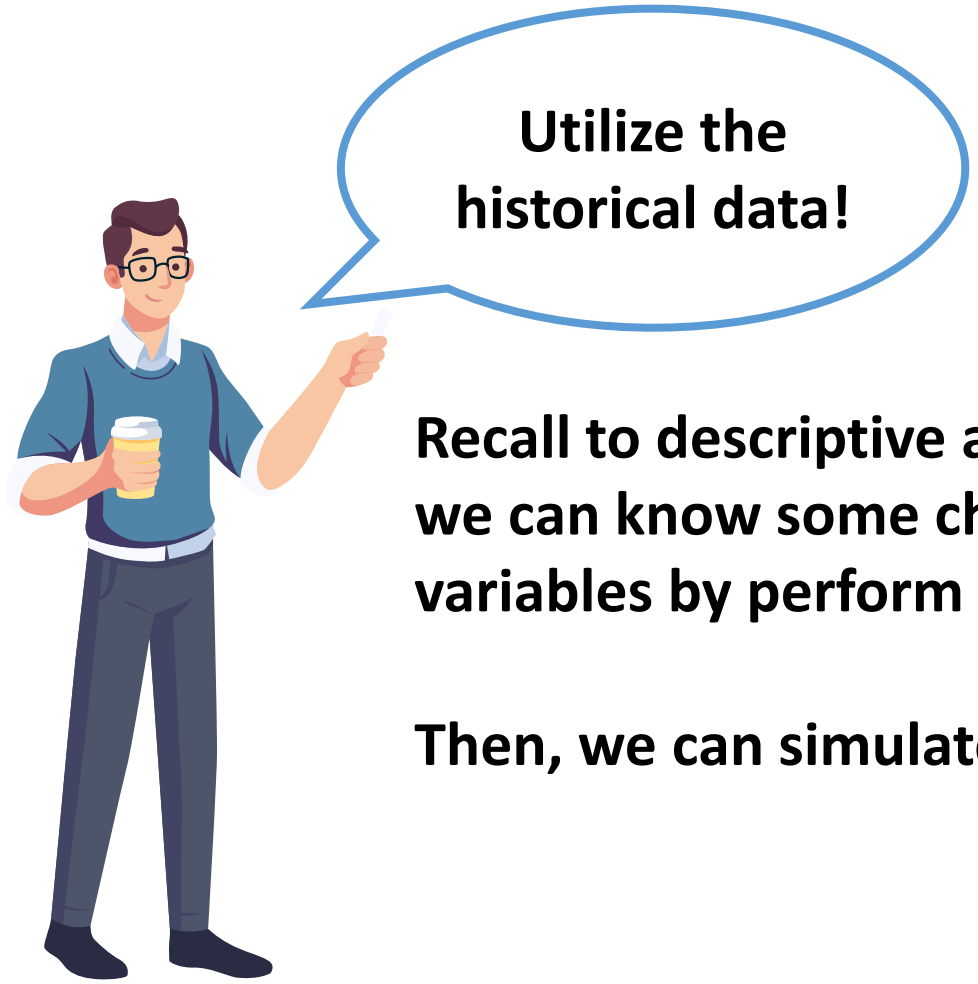
- Counter a customer picks → Random under uniform distribution

Expert can tell us what the simulation should be.

If I don't have any expert for the system that I want to construct.

Simulation

Prescriptive Analysis



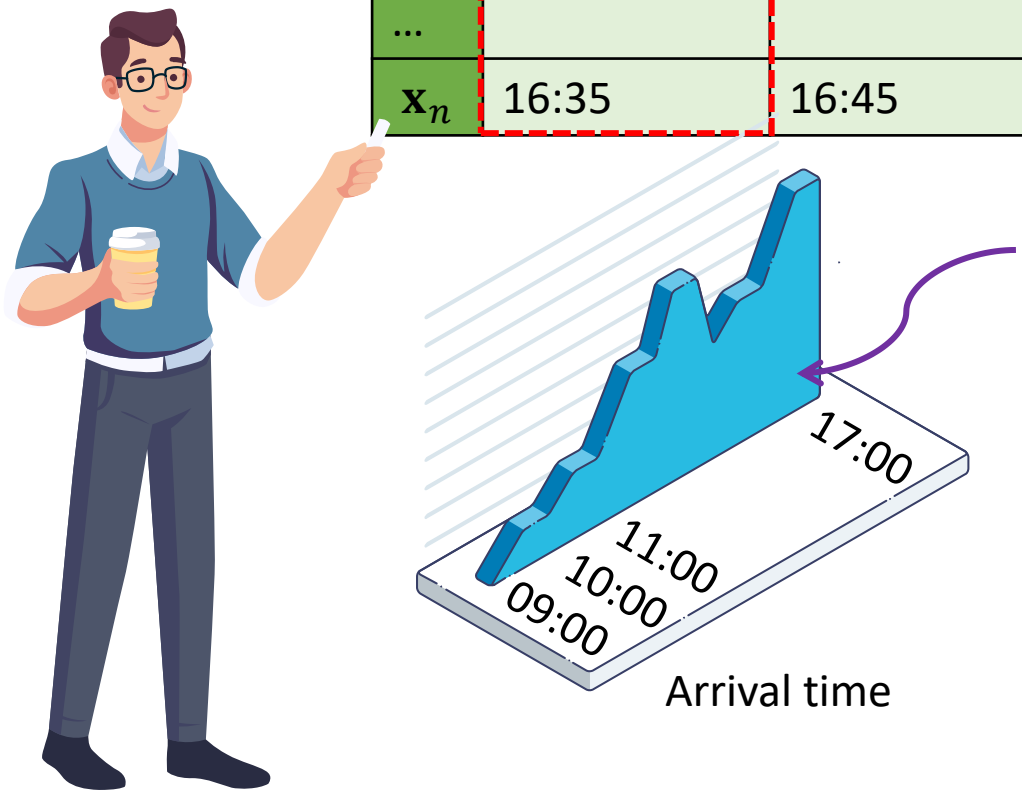
Recall to descriptive analysis,
we can know some characteristics/behavior of random variables by perform descriptive analysis on historical data.

Then, we can simulate a system under the behavior.

Simulation

Prescriptive Analysis

	Arrival time	Service time	End service time	Counter No.
$\mathbf{x}_1$	9:30	9:05	9:20	1
$\mathbf{x}_2$	9:33	9:21	9:30	1
...				
$\mathbf{x}_n$	16:35	16:45	17:00	4



We can generate random numbers under the distribution.

Simulation

Prescriptive Analysis

Monte Carlo simulation

Computerized mathematical technique that allows people to account for risk in quantitative analysis and decision making.

Monte Carlo Casino, Monaco.

Simulation

Prescriptive Analysis

Pattern of Monte Carlo simulation process

1. Define a domain of possible inputs
2. Generate inputs randomly from a probability distribution over the domain

3. Perform a deterministic computation on the inputs
4. Aggregate the results

Simulation

Prescriptive Analysis

Example: Approximating the value of π

1	2	3	4
Define a domain of possible inputs	Generate inputs randomly from a probability distribution over the domain	Perform a deterministic computation on the inputs	Aggregate the results
(x,y) - The values of x and y are in [-1,1]	Random values of x and y are in [0,1] under uniform distribution.	Count the number of points inside the circle.	Approximate the value of π by: $\pi \approx \frac{4 \times N_{inner}}{N_{tatal}}$

See animation at https://academo.org/demos/estimating-pi-monte-carlo/

Simulation

Prescriptive Analysis

Applying Monte Carlo simulation to decision making

Example: Monty Hall Problem

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats.

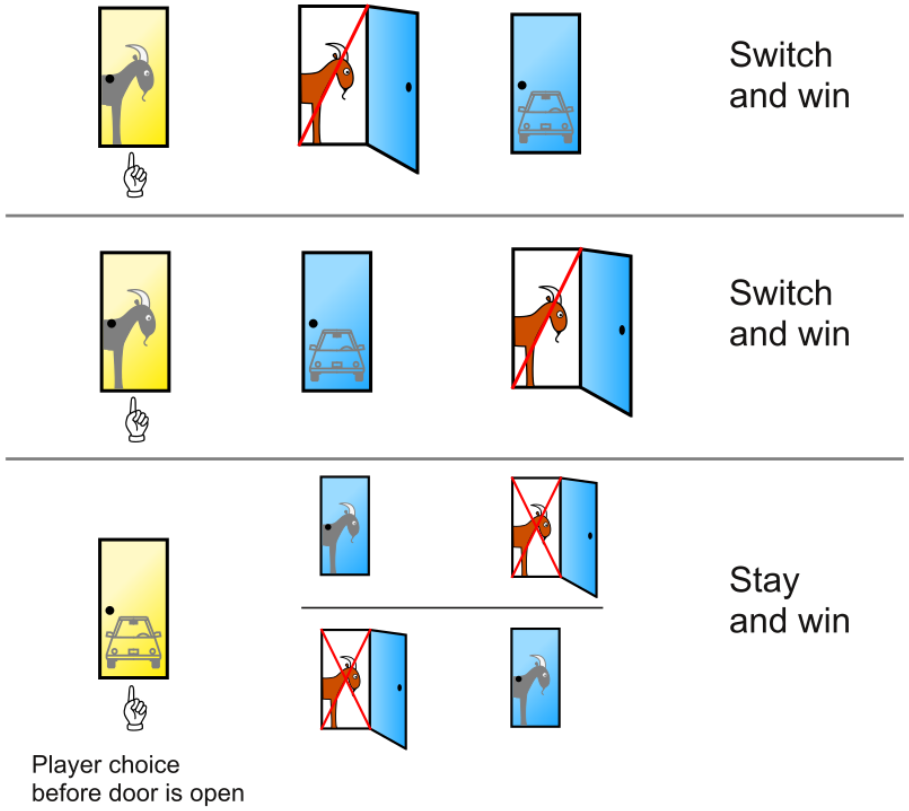
- You pick a door, say No. 1,
- The host, who knows what's behind the doors, opens another door, say No. 3, which has a goat.
- He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

Simulation

Prescriptive Analysis

Applying Monte Carlo simulation to decision making

Example: Monty Hall Problem



Three initial configurations of the game.
In two of them, the player wins by switching away from the choice made before a door was opened.

Source
https://en.wikipedia.org/wiki/Monty_Hall_problem#/media/File:Monty_Hall_Problem_-_Standard_probabilities.svg

Simulation

Prescriptive Analysis

Applying Monte Carlo simulation to decision making

Simulating Monty Hall with Monte Carlo

1	2	3	4
Define a domain of possible inputs	Generate inputs randomly from a probability distribution over the domain	Perform a deterministic computation on the inputs	Aggregate the results
***N* Doors** – each door has either car or goat, w.r.t. only one door has car. **Initial door** – select 1 from N doors. **Decision** – stay or switch.	Random to put car behind a door. Random the initial door. Random your decision.	Perform *M* times Count the number of success on switch, S_{switch}. Count the number of success on non-switch, S_{stay}.	Calculate Prob. of success if we do not switch doors. Calculate Prob. of success if we switch doors.

Simulation

Prescriptive Analysis

Applying Monte Carlo simulation to decision making

Simulating Monty Hall with Monte Carlo

After we simulate 10000 games of Monty Hall with 3 doors

We got the results as follows:
Number of success on switch: 666566 66.6566%
Number of success on non-switch: 333434 33.3434%

We can see that the success percentages don't vary much, and it tells us that if we make the switch then the chances of winning are 2 out of 3 times.

Further Study

Papers:

- Katerina Lepenioti, Alexandros Bousdekis, Dimitris Apostolou, Gregoris Mentzas, Prescriptive analytics: Literature review and research challenges, International Journal of Information Management, Volume 50, 2020, Pages 57-70, https://doi.org/10.1016/j.ijinfomgt.2019.04.003.
- Lepenioti K., Bousdekis A., Apostolou D., Mentzas G. (2019) Prescriptive Analytics: A Survey of Approaches and Methods. In: Abramowicz W., Paschke A. (eds) Business Information Systems Workshops. BIS 2018. Lecture Notes in Business Information Processing, vol 339. Springer, Cham

Website

- https://towardsdatascience.com/every-data-scientist-needs-to-read-these-simulation-stories-7be0531e782f
- https://towardsdatascience.com/a-zero-math-introduction-to-markov-chain-monte-carlo-methods-dcba889e0c50