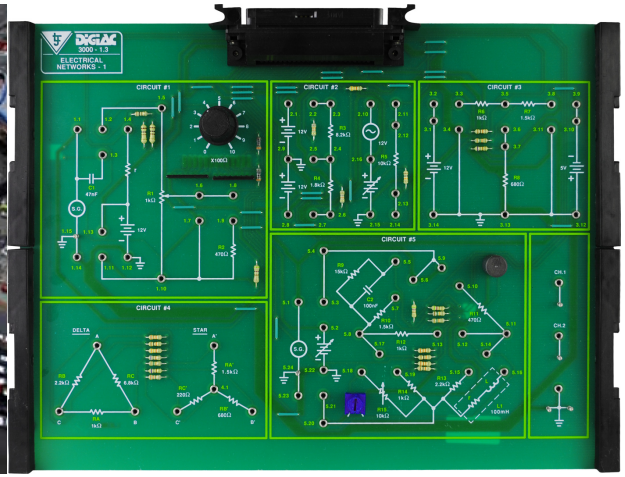
# 201110 Intrgrated Math SC

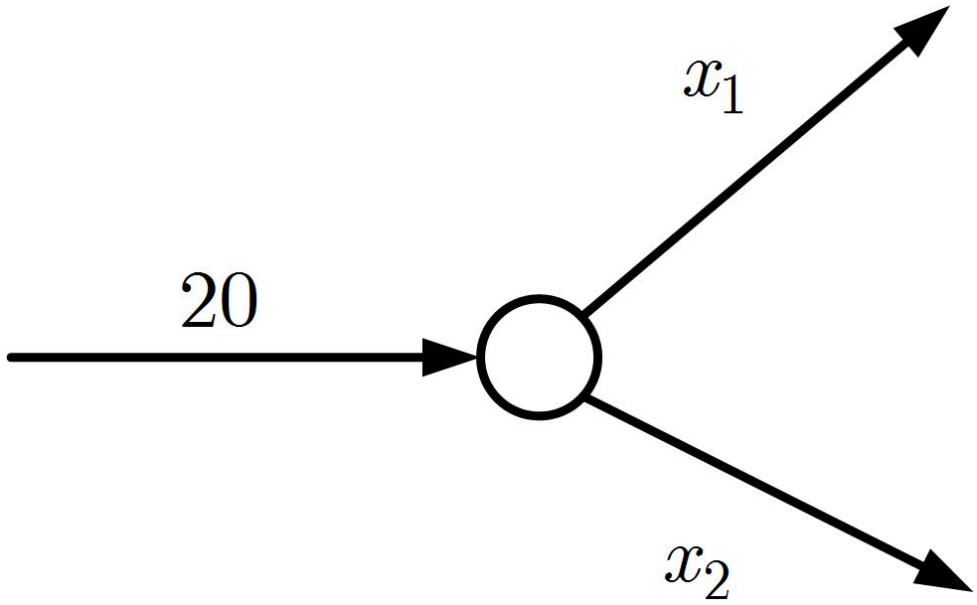
Faculty of Science
Chiang Mai University

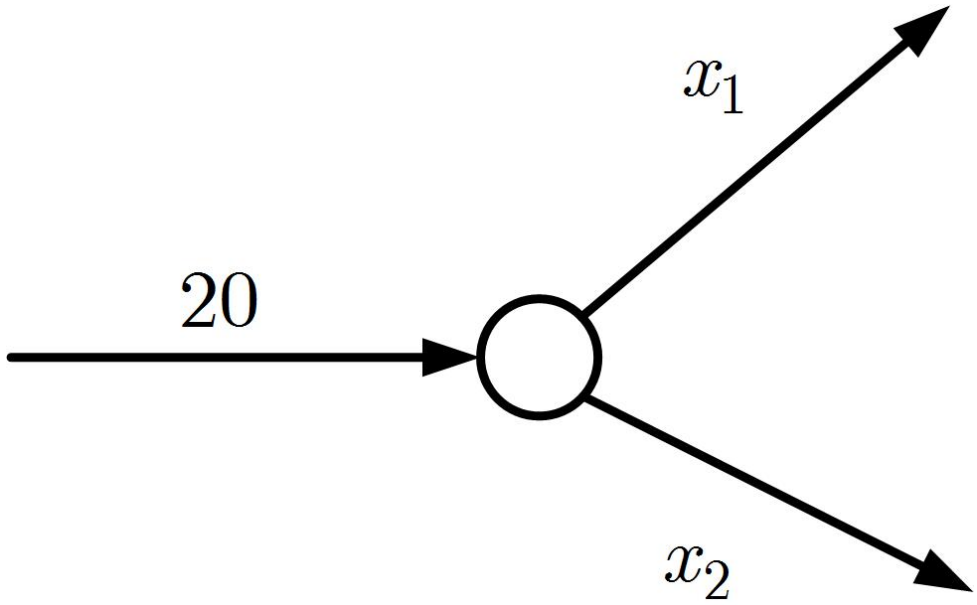
## Matrix Applications

# Network Analysis

- Traffic Flow
- Electrical Networks

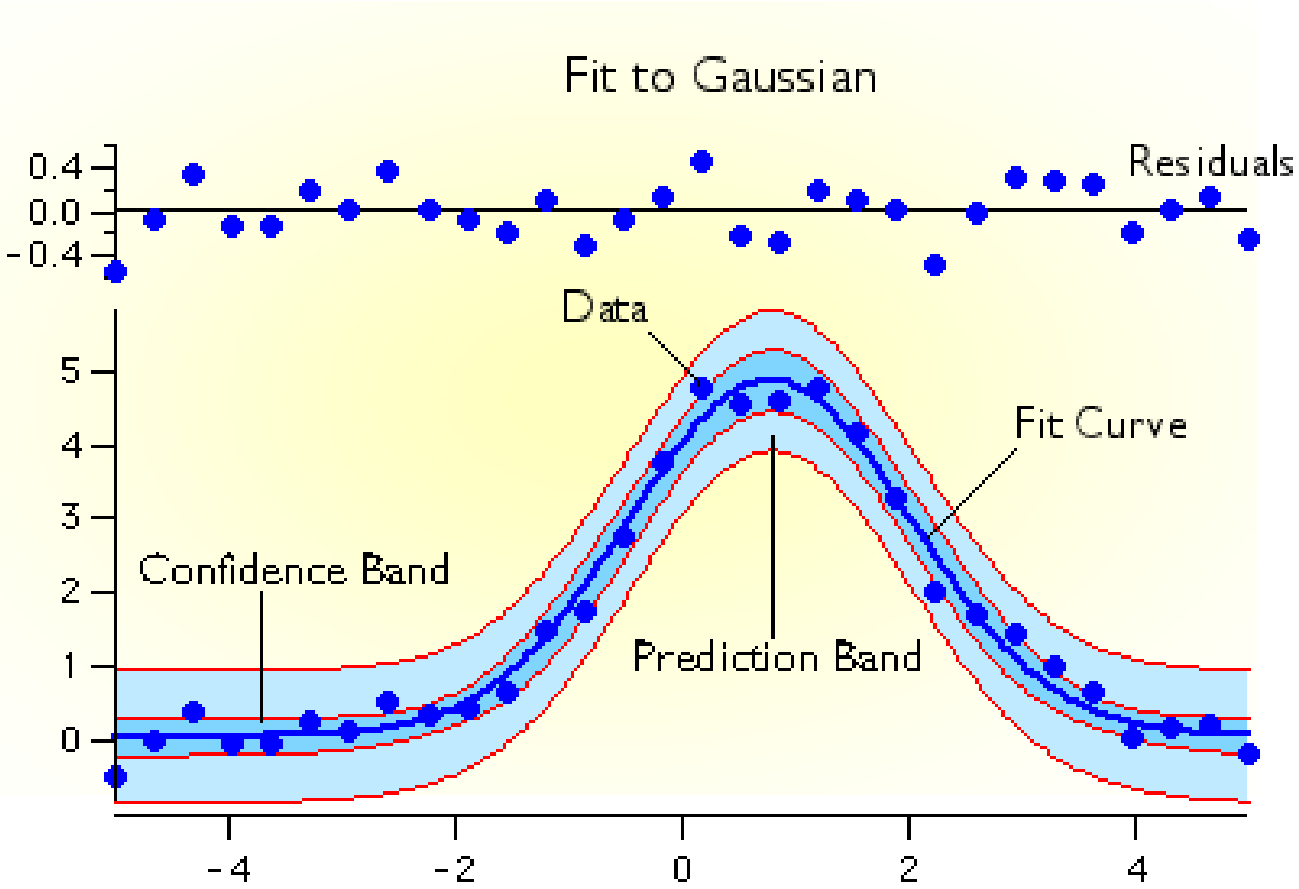



$x_1$
20
$x_2$

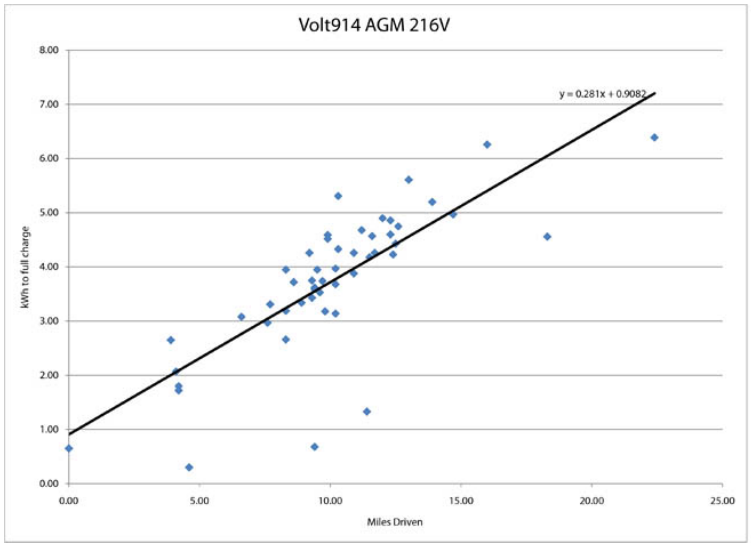


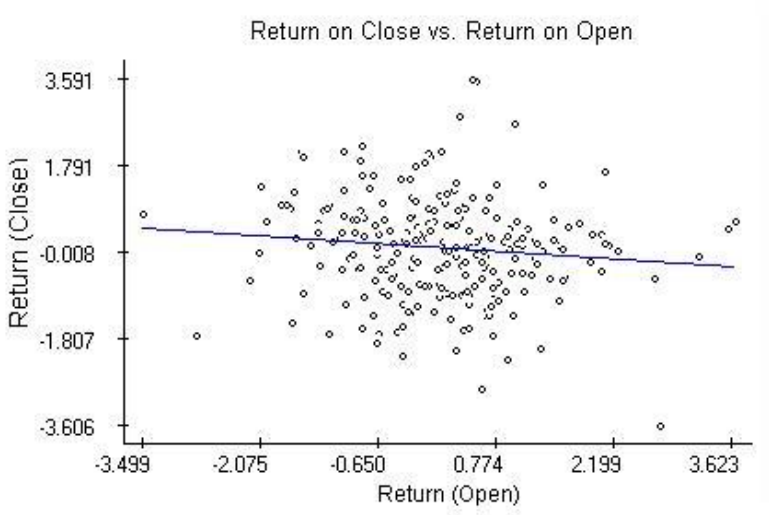

$$20 = x_1 + x_2$$

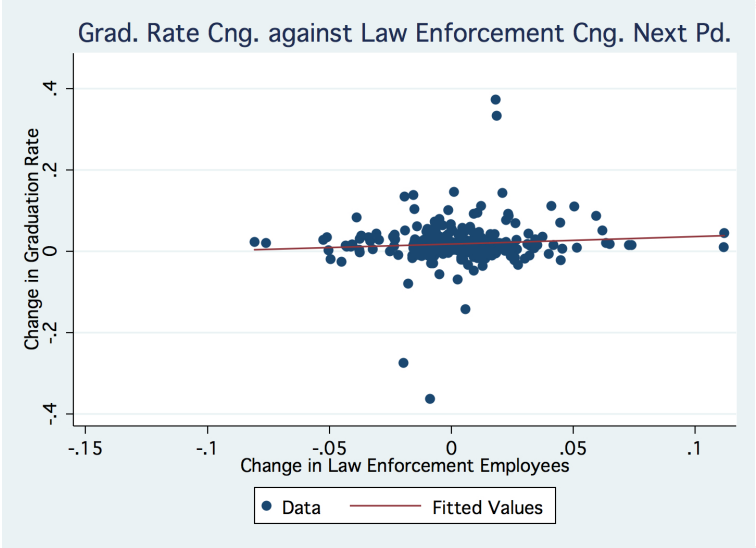
# Least – Square Curves


Fit to Gaussian
Residuals
0.4
0.0
-0.4
Data
Fit Curve
Confidence Band
Prediction Band
5
4
3
2
1
0
-4
-2
0
2
4

# Least – Square Lines


Volt914 AGM 216V
y = 0.281x + 0.9082
kWh to full charge
Miles Driven
0.00
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
0.00
5.00
10.00
15.00
20.00
25.00


Return on Close vs. Return on Open
Return (Close)
Return (Open)
3.591
1.791
-0.008
-1.807
-3.606
-3.499
-2.075
-0.650
0.774
2.199
3.623


Grad. Rate Cng. against Law Enforcement Cng. Next Pd.
Change in Graduation Rate
Change in Law Enforcement Employees
-.4
-.2
0
.2
.4
-.15
-.1
-.05
0
.05
.1
Data
Fitted Values

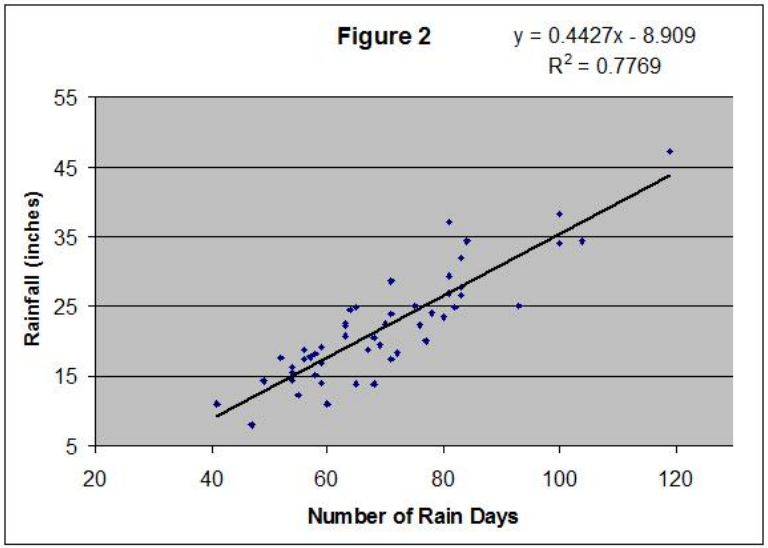
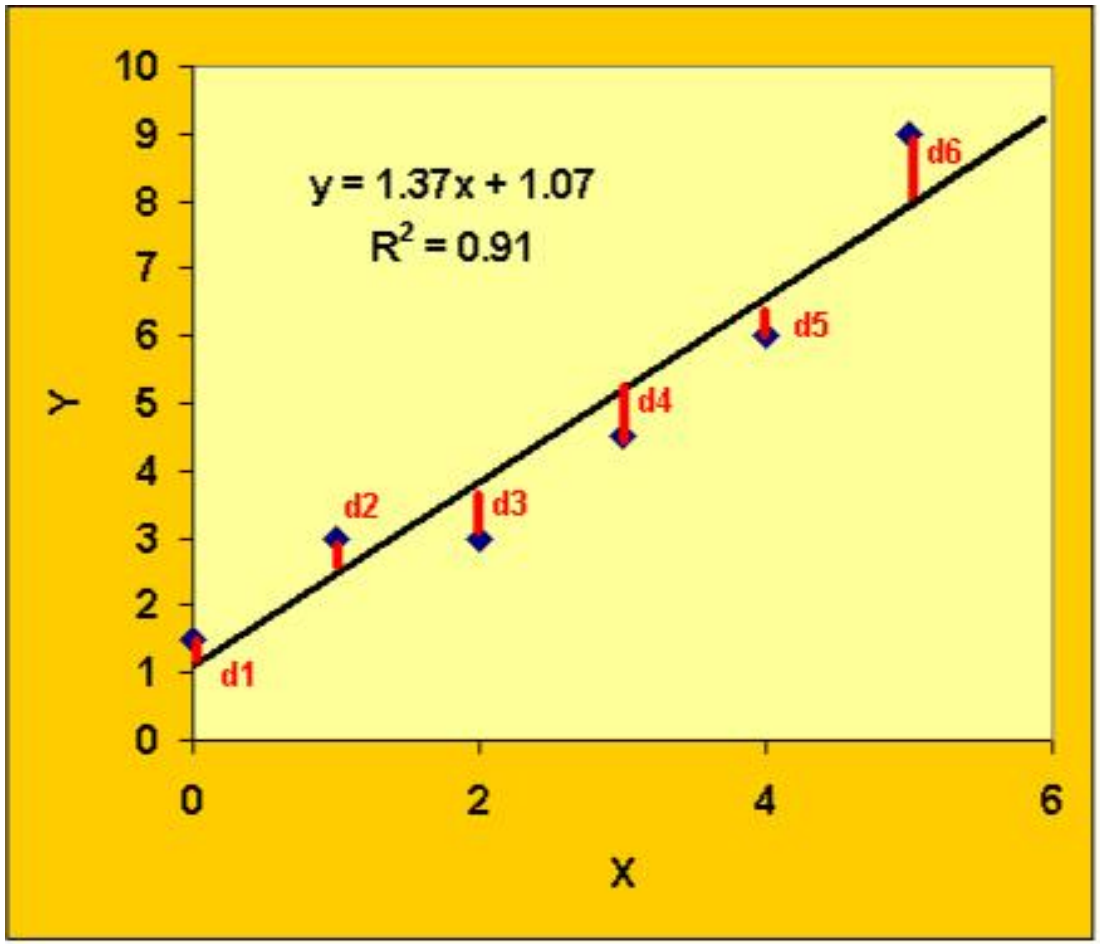


Figure 2
y = 0.4427x - 8.909
R2 = 0.7769
Rainfall (inches)
Number of Rain Days
5
15
25
35
45
55
20
40
60
80
100
120


y = 1.37x + 1.07
R2 = 0.91
d1
d2
d3
d4
d5
d6
Y
X
0
1
2
3
4
5
6
7
8
9
10
0
2
4
6

Given the informations

$$(x_1, y_1), \quad (x_2, y_2), \quad (x_3, y_3)$$

Finding a least square line, we can write a linear system by

$$\begin{aligned} y_1 &= a + bx_1 \\ y_2 &= a + bx_2 \\ y_3 &= a + bx_3 \end{aligned}$$

want to find $\boxed{a,\ b}$

$$y_1 = a + bx_1$$
$$y_2 = a + bx_2$$
$$y_3 = a + bx_3$$

Rewrite a linear system into matrix form

$$X = Y$$

by

$$X =$$

$$y_1 = a + bx_1$$
$$y_2 = a + bx_2$$
$$y_3 = a + bx_3$$

Rewrite a linear system into matrix form

$$X = Y$$

by

$$X = \begin{pmatrix} a \\ b \end{pmatrix},$$

$$y_1 = a + bx_1$$
$$y_2 = a + bx_2$$
$$y_3 = a + bx_3$$

Rewrite a linear system into matrix form

$$X = Y$$

by

$$X = \begin{pmatrix} a \\ b \end{pmatrix}, \quad A =$$

$$y_1 = a + bx_1$$
$$y_2 = a + bx_2$$
$$y_3 = a + bx_3$$

Rewrite a linear system into matrix form

$$X = Y$$

by

$$X = \begin{pmatrix} a \\ b \end{pmatrix}, \quad A = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \end{pmatrix},$$

$$y_1 = a + bx_1$$
$$y_2 = a + bx_2$$
$$y_3 = a + bx_3$$

Rewrite a linear system into matrix form

$$X = Y$$

by

$$X = \begin{pmatrix} a \\ b \end{pmatrix}, \quad A = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \end{pmatrix}, \quad Y =$$

$$y_1 = a + bx_1$$
$$y_2 = a + bx_2$$
$$y_3 = a + bx_3$$

Rewrite a linear system into matrix form

$$X = Y$$

by

$$X = \begin{pmatrix} a \\ b \end{pmatrix}, \quad A = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \end{pmatrix}, \quad Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

Normally

$$AX = Y$$
$$X = A^{-1}Y$$

Normally

$$AX = Y$$
$$X = A^{-1}Y$$

can not find $A^{-1}$, because $A^{-1}$ is not a square matrix.

$$A = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \end{pmatrix}$$

# Pesudo Inverse

$$AX = Y$$
$$A^TAX = A^TY$$
$$X = (A^TA)^{-1}A^TY$$

Therefore, we can find $X = \begin{pmatrix} a \\ b \end{pmatrix}$ from

$$X = (A^TA)^{-1}A^TY$$

where $(A^TA)^{-1}A^T$ is a **pesudo inverse.**

# Example

Finding a least square line from $(1, 1)$, $(2, 2)$, $(3, 4)$.