201110 Intrgrated Math SC

Faculty of Science
Chiang Mai University

Matrix Operations

Contents

- Matrix Multiplication
- Matrix Addition
- Matrix Substraction
- Matrix Inverstion
- Solving a Linear System

Matrix Multiplication

- The Product of Two vectors
- The Product of a matrix and a vector
- The Product of two matrices

$$\boxed{\text{Matrix } (a * b) \text{ times matrix } (b * c) \implies \text{matrix } (a * c)}$$

- Matrix Addition
- Matrix Substraction

Matrix Inversion

If A is an $n \times n$ inveritble matrix, then

$$A^{-1} = \frac{1}{\det(A)} \text{adj}(A).$$

System of Linear Equations

System of linear equations -- m–equations, n–dimension

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\
\vdots \quad & \vdots \quad \vdots \quad \vdots \quad \vdots \\
a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &= b_m
\end{aligned}
$$

can be rewritten in matrix form

$$
\begin{pmatrix}
a_{11} & a_{12} & \ldots & a_{1n} \\
a_{21} & a_{22} & \ldots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{m1} & a_{m2} & \ldots & a_{mn}
\end{pmatrix}
\begin{pmatrix}
x_1 \\ x_2 \\ \ldots \\ x_n
\end{pmatrix}
=
\begin{pmatrix}
b_1 \\ b_2 \\ \ldots \\ b_m
\end{pmatrix}
\Longrightarrow
$$

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\begin{pmatrix}
a_{11} & a_{12} & \ldots & a_{1n} \\
a_{21} & a_{22} & \ldots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{m1} & a_{m2} & \ldots & a_{mn}
\end{pmatrix}
\begin{pmatrix}
x_1 \\ x_2 \\ \ldots \\ x_n
\end{pmatrix}
=
\begin{pmatrix}
b_1 \\ b_2 \\ \ldots \\ b_m
\end{pmatrix}
\Longrightarrow AX = B
$$

Solution set

A linear system may behave in any one of three possible ways:

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1. The system has infinitely many solutions.
2. The system has a single unique solution.
3. The system has no solution.

Solving a Linear System

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- Elimination of variables

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- Row reduction (Gaussian elimination)

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- Cramer's rule

Solving a Linear System

- Elimination of variables
- Row reduction (Gaussian elimination)
- Cramer's rule
- Matrix solution