Simulation

from Wikipedia and Youtube

Simulation

- **Simulation** is the imitation of the operation of a real-world process or system over time. Can be physical, or mathematical

System

- A set of interacting components or entities operating together
- Examples:
  - Traffic of Chiang Mai with roads, traffic policy, and vehicles
  - A stock markets with traders, regulators and stock exchange
  - The solar system with the Sun and its planets
  - A chemical process, with the interacting chemicals and the environment

Components of A System

- **Entity**: object of interest in the system.
- **Attribute**: characteristic of all entities, can vary from one entities to another.
- **Resources**: what entities compete for, having them allow entities to do something.
- **Variable**: a piece of information that tell us something about the system
- **State**: a collection of variables that contains all the information necessary that describe that system at any time.
- **Event**: An instantaneous occurrence that changes the state of the system.
- **Activity**: represents a time period of specify length

Example of a System: a Football Game

- **Entity**: players from each team, the referee.
- **Attribute**: accuracy, speed, strength of each player.
- **Resources**: the ball, allowing player to score (shoot the ball through the goal).
- **Variable**: condition, location of each player, referee, etc.
- **State**: {locations of players, location of the ball, who got the ball, current score…}
- **Event**: goal, own-goal, red-card, etc.
- **Activity**: first half, second half, procession of ball, etc.

What is Simulation For?

- To ask questions about system that is too complex to solve mathematically, or too expensive to experiment physically.
  - “Will the traffic be less congested if we make Huay Kaew road one-way?”
  - “If current humidity is 90%, how likely it is to rain in 6 hours?”
- Those answer can be used in:
  - Decision making
    - Simulation can show the result of the decision beforehand
    - Including planning ahead for emergency
  - System analysis
    - How a component will behave if a parameter is modified
  - System synthesis
    - Create a better system with the help of simulation result

Physical simulation and Interactive simulation

- **Physical simulation** refers to simulation in which physical objects (physical model) are substituted for the real thing based on laws of physics.
- **Interactive simulation** is a special kind of physical simulation in which physical simulations include human operators such as in a flight simulator or a driving simulator.

Cockpit of a flight simulator.

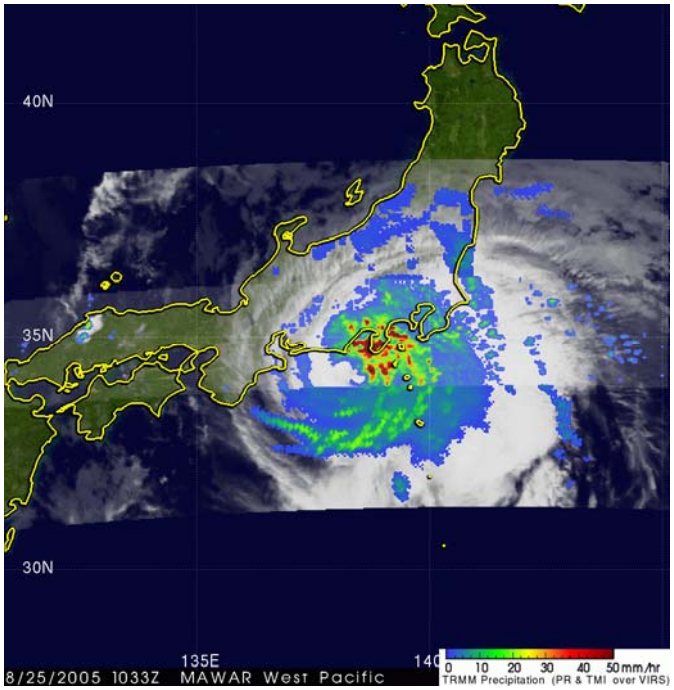
Example: A Tsunami Simulator

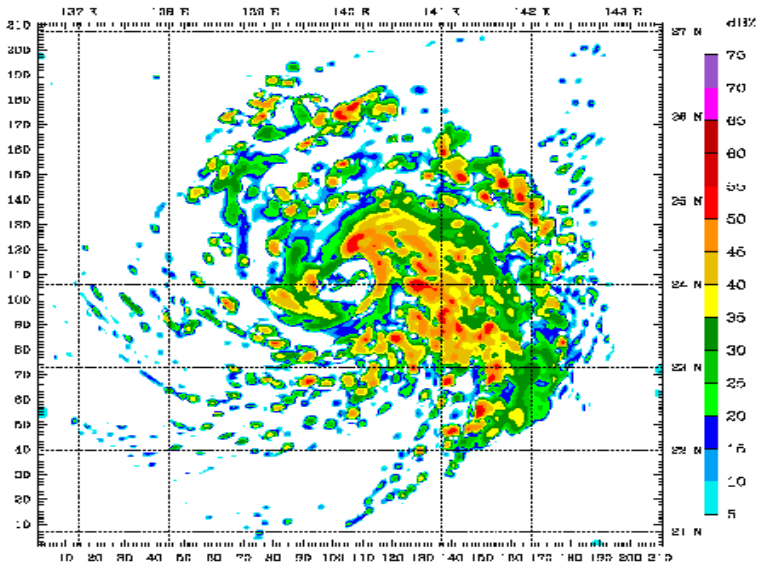
https://www.youtube.com/watch?v=Ez_TOJe-oaA

Computer simulation

- A **computer simulation** is a simulation, run on computers to reproduce behavior of a system.
- The simulation uses an abstract model (a **computational model** or sometime called **mathematical model**) to simulate the system.
- Computer simulation has become a useful part of mathematical modeling many natural systems in **physics**, **astrophysics**, **climatology**, **chemistry**, **biology**, and human systems such as **economics**, **psychology**, **social science**, and **engineering**.

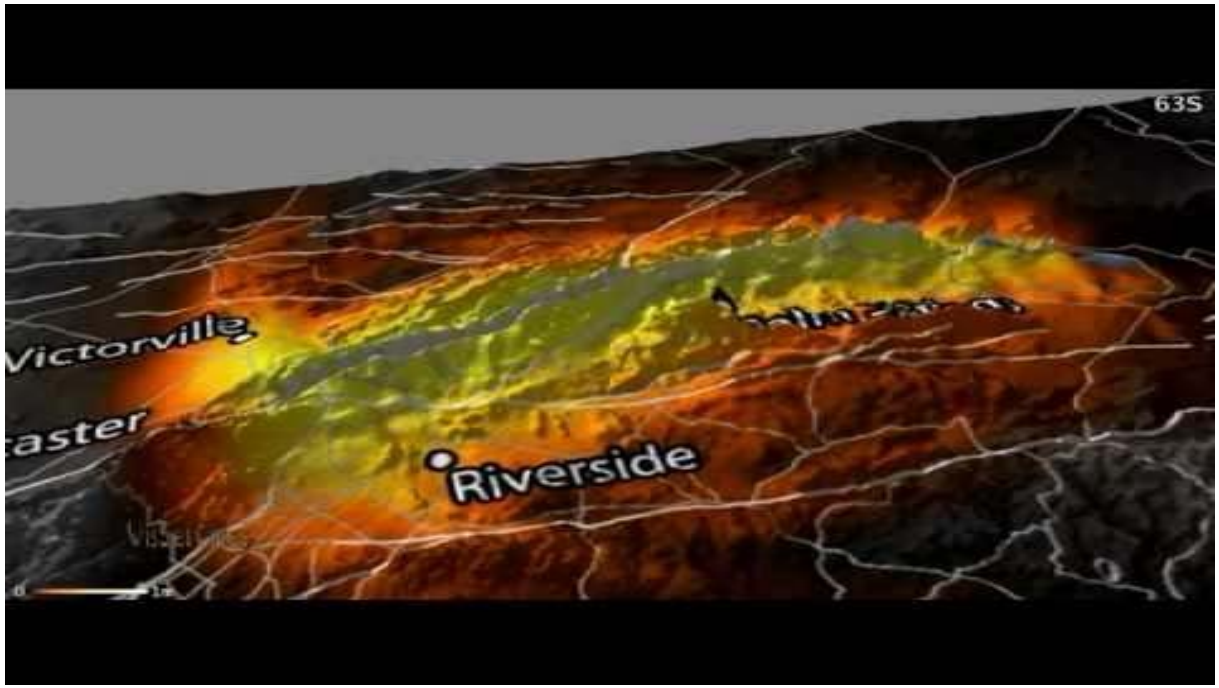
Simulation of Typhoon





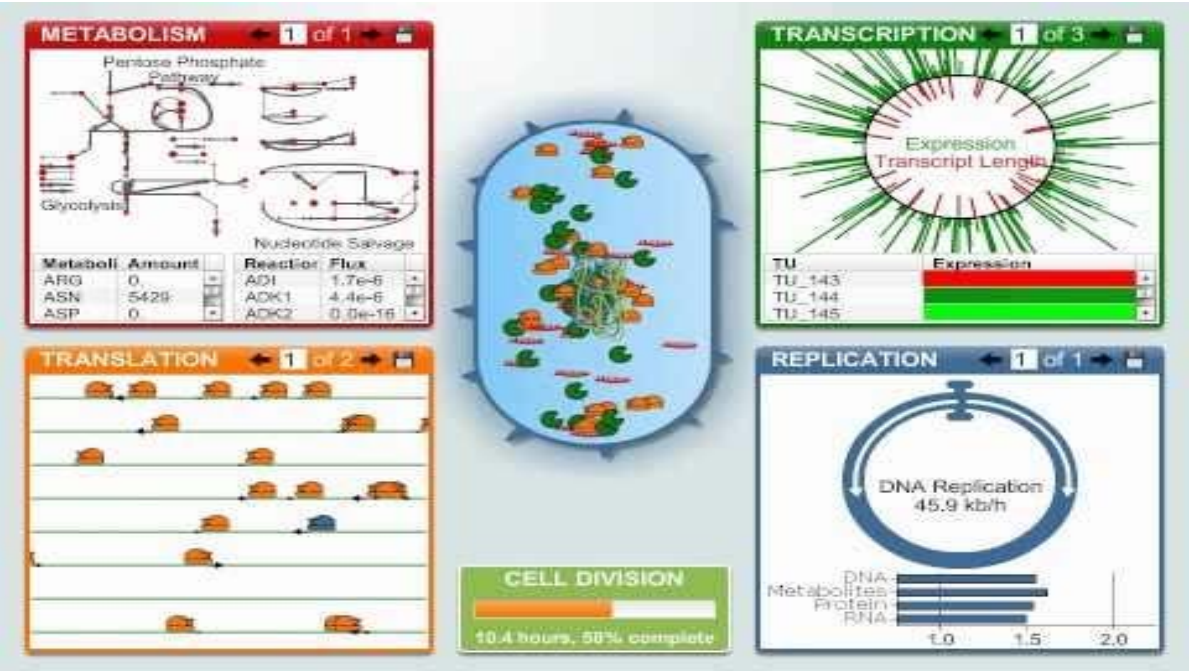
Simulation of Typhoon Mawar, Japan, 2005

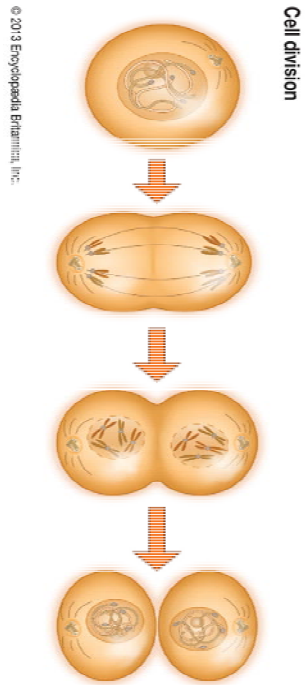
Earthquake Simulations



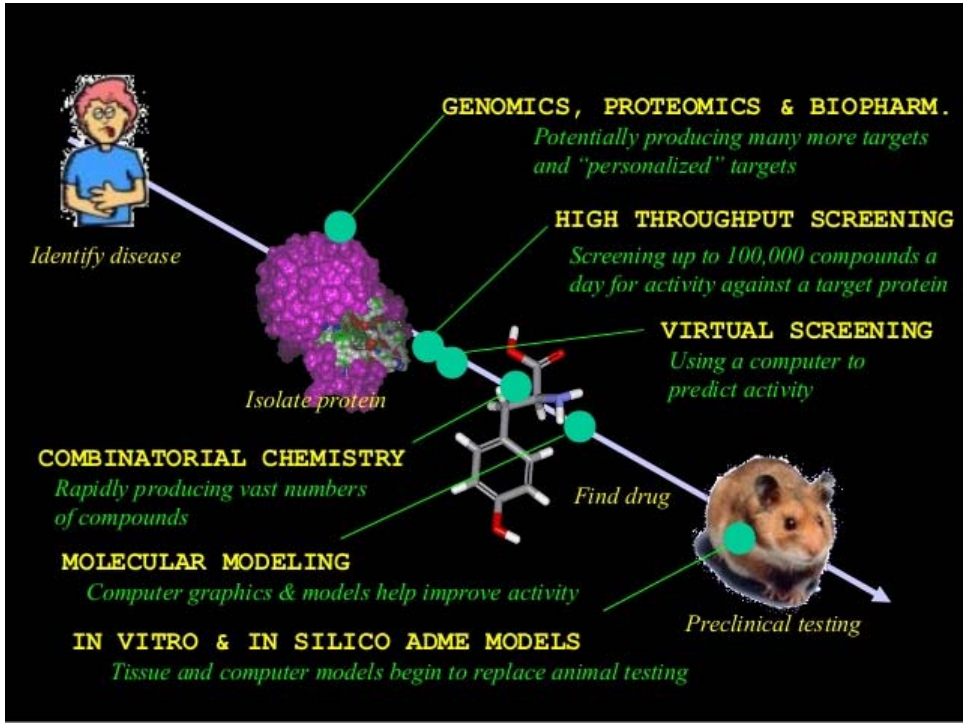
San Andreas Fault, Southern California, USA

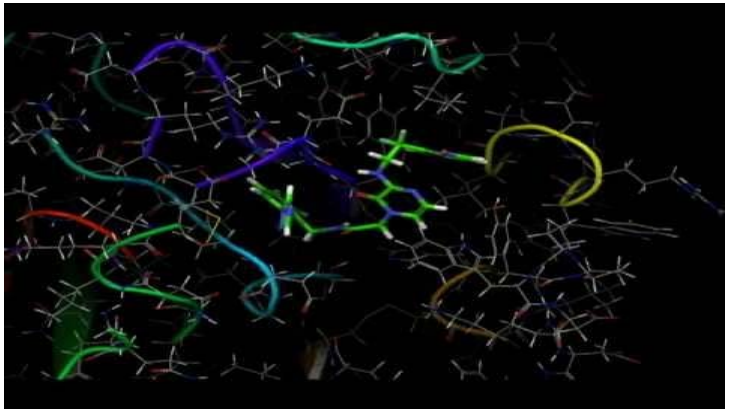
Life Cycle of the First In silico Cell



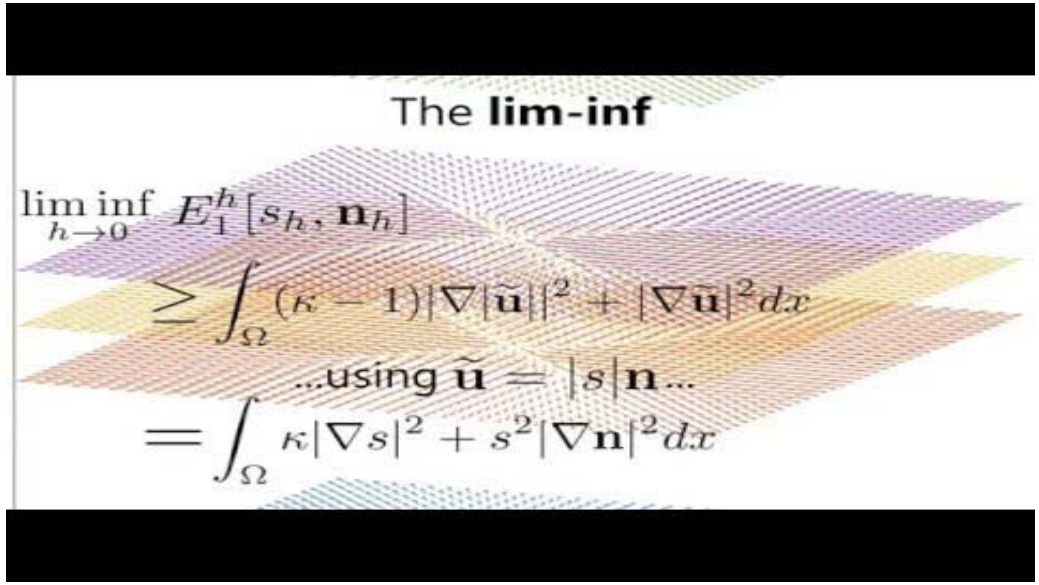


Computational Chemistry for Drug Design

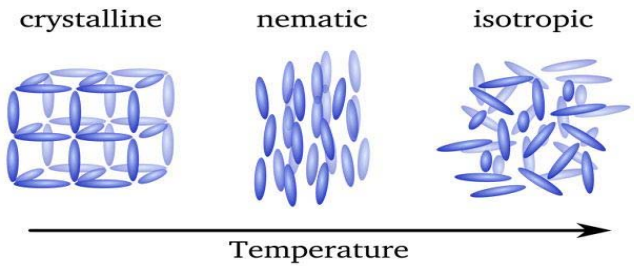




Mathematical Models and Simulation in Material Science

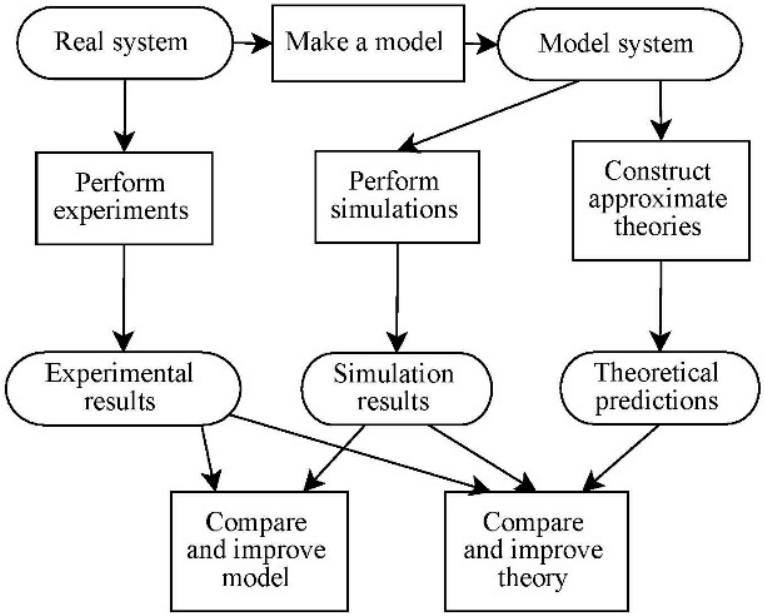


Simulation of Nematic Liquid Crystals

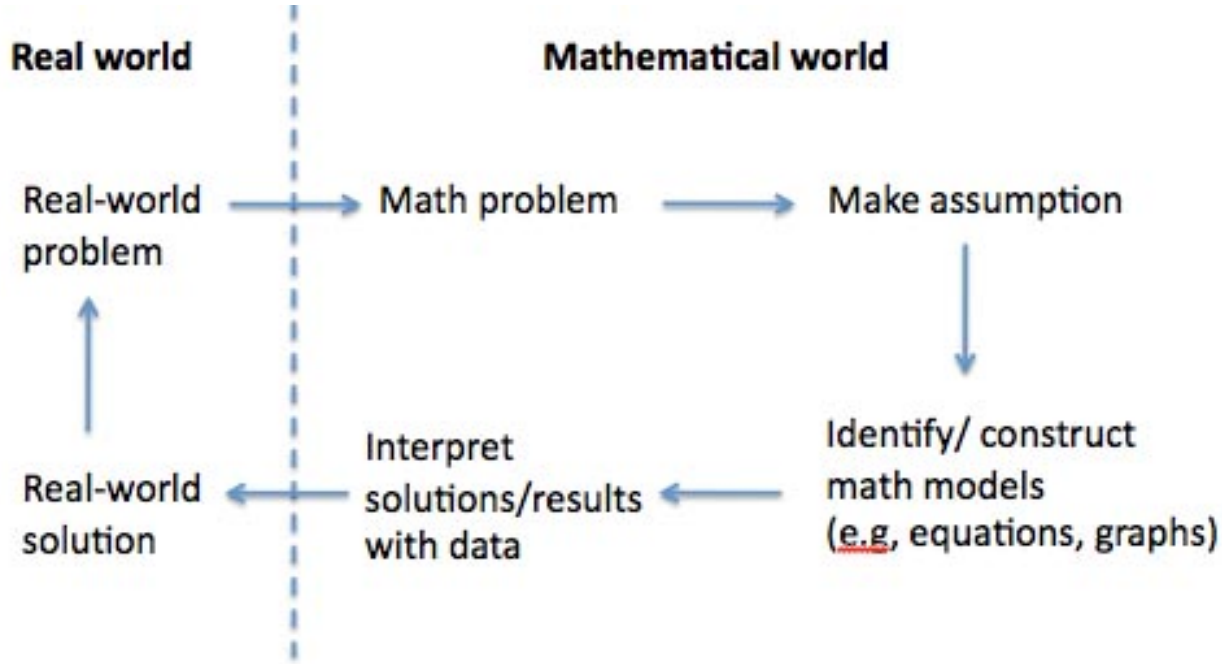


Simulation Model

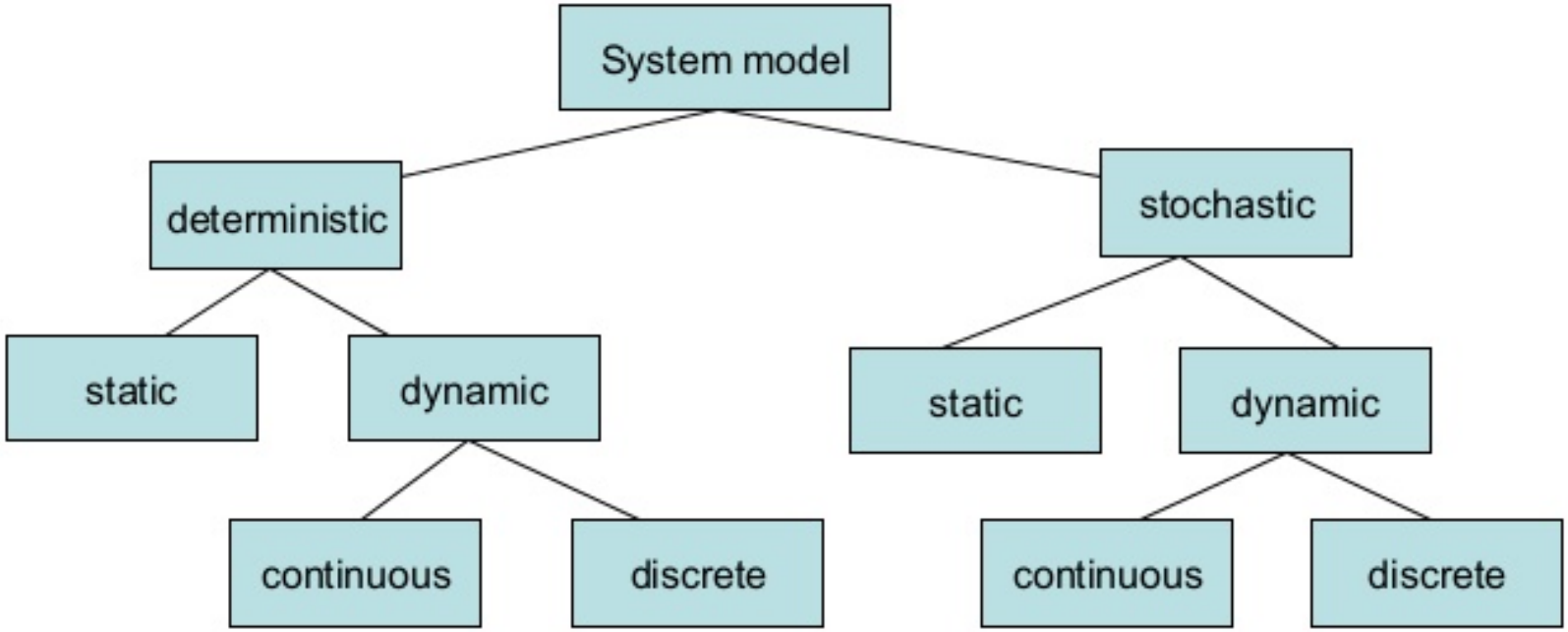
- **Simulation model** represents the key characteristics or behaviors/functions of the selected physical or abstract system or process.



Mathematical Modeling



Types of Mathematical Models



Continuous-Time and Discrete-Time Mathematical Models

Continuous-Time Systems

Most continuous time systems represent how continuous signals are transformed via **differential equations**.

E.g. circuit, car velocity

$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} v_s(t)$$

$$m\frac{dv(t)}{dt} + \rho v(t) = f(t)$$

First order differential equations

Discrete-Time Systems

Most discrete time systems represent how discrete signals are transformed via **difference equations**

E.g. bank account, discrete car velocity system

$$y[n] = 1.01y[n-1] + x[n]$$

$$v[n] - \frac{m}{m+\rho\Delta} v[n-1] = \frac{\Delta}{m+\rho\Delta} f[n]$$

$$\frac{dv(n\Delta)}{dt} = \frac{v(n\Delta) - v((n-1)\Delta)}{\Delta}$$

Differential Equations (Continuous Time Models)

- A **differential equation** is a mathematical equation that relates some

ODE (ORDINARY DIFFERENTIAL EQUATION):

An equation contains only ordinary derivates of one or more dependent variables of a single independent variable.

For Example,

$dy/dx + 5y = e^x$, $(dx/dt) + (dy/dt) = 2x + y$

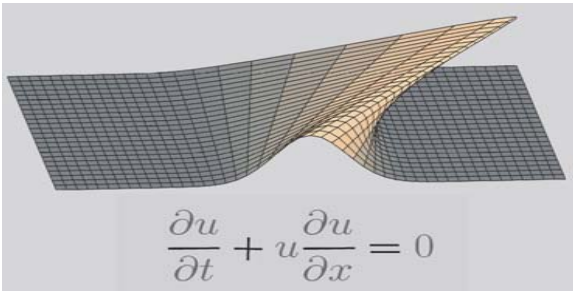
S(t)= $\frac{t^2}{2}\frac{d^2s}{dt^2} + t\frac{ds}{dt} + s_0$

PDE (PARTIAL DIFFERENTIAL EQUATION):

An equation contains partial derivates of one or more dependent variables of two or more independent variables.

For Example,

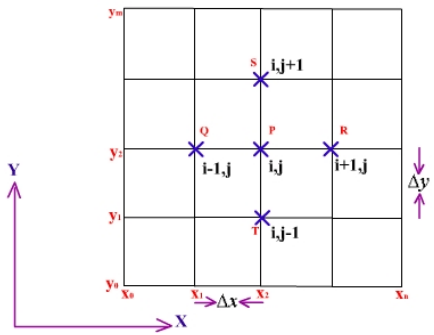
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial t^2} = 2\frac{\partial u}{\partial t} \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

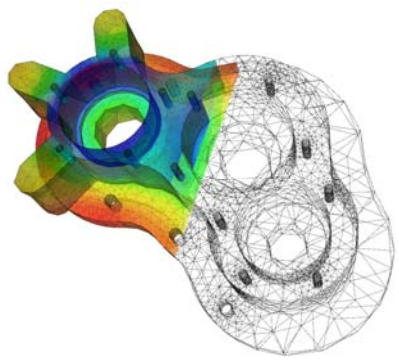


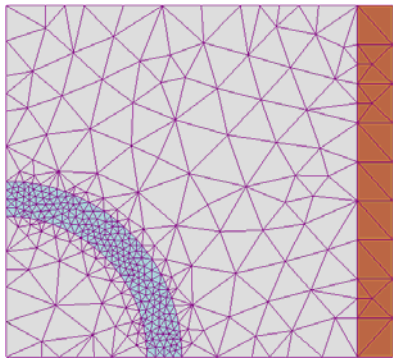
Difference Equations (Discrete-Time Models)

- The **finite-difference methods** are numerical methods for solving **differential equations** by approximating them with difference equations, in which finite differences approximate the derivatives.

- The **finite element methods** are numerical techniques for finding approximate solutions to boundary value problems for **partial differential equations**.



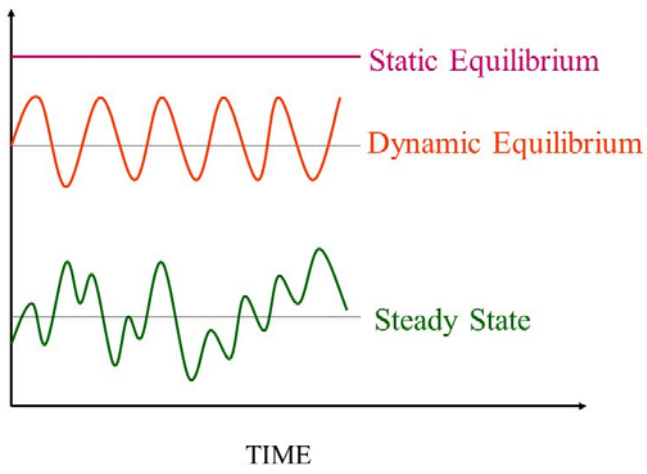




Static and Dynamic Models

STATIC MODEL A static (or steady-state) model calculates the system in equilibrium, and thus is time-invariant. A static model cannot be changed, and one cannot enter edit mode when static model is open for detail view.

DYNAMIC MODEL A dynamic model accounts for time-dependent changes in the state of the system. Dynamic models are typically represented by differential equations.



Static Models

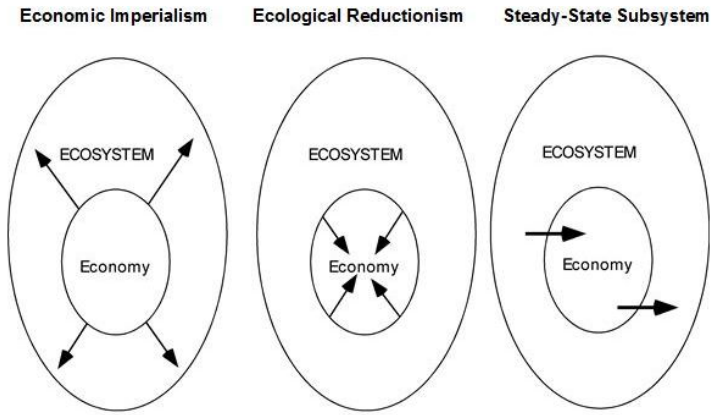
- A system is in a **steady state** if the variables which define the behavior of the system or the process are unchanging in time.
- In continuous time, this means that for those properties *p* of the system, the partial derivative with respect to time is zero.

$$\frac{\partial p}{\partial t} = 0 \qquad \text{for all } t$$

- In discrete time, it means that the first difference of each property is zero.

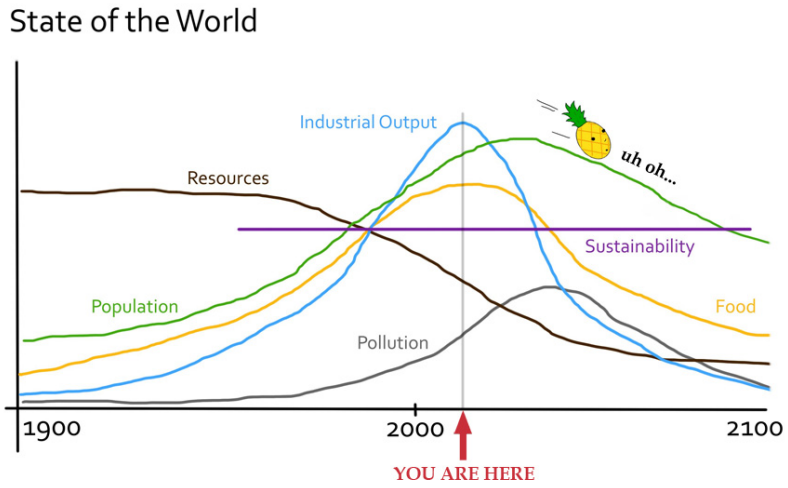
$$p_{t+1} - p_t = 0 \qquad \text{for all } t$$

Economic System



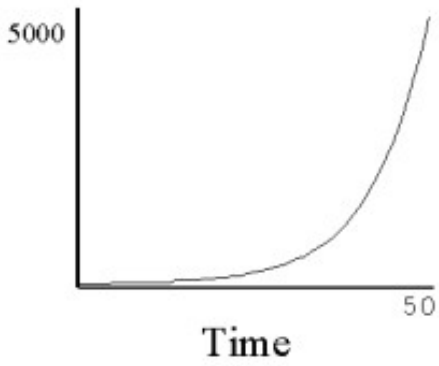
Dynamic Models

- A **dynamic model** represents the behaviour of a system over time.
- It is used where the system's behaviour is best described as a set of states that occur in a sequence of time.
- The components of the **dynamic model** are: **States**.

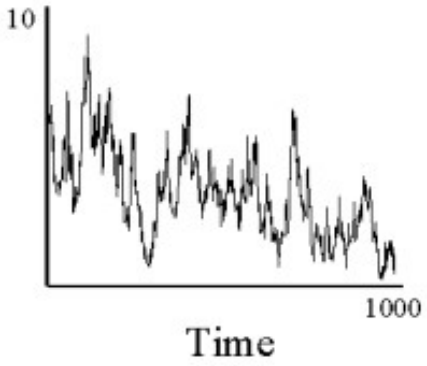


Deterministic and Stochastic Models

- In deterministic models, the output of the model is fully determined by the parameter values and the initial conditions.
- Stochastic models possess some inherent randomness. The same set of parameter values and initial conditions will lead to an ensemble of different outputs.



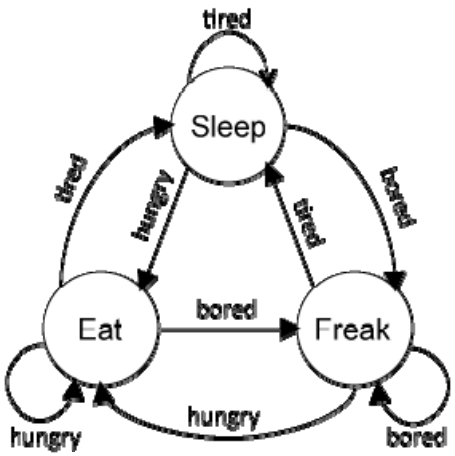
Continuous Deterministic



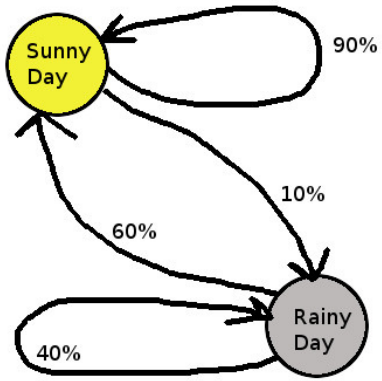
Continuous Stochastic

Deterministic Models

- A **deterministic system** is a system in which no randomness is involved in the development of future states of the system.
- A deterministic model will always produce the same output from a given starting condition or initial state.



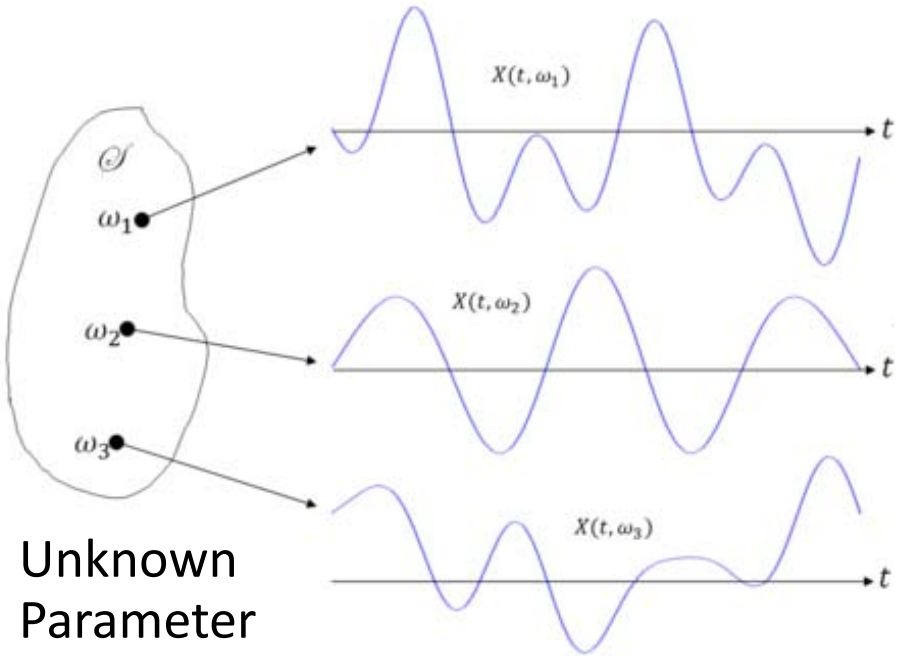
Finite State Machine
(Discrete Deterministic Model)



Markov Chain
(Discrete Stochastic Model)

Stochastic Process

- A **stochastic process** is a random process evolving with time.
- The evolution of the stochastic system is represented by a set of random variables.



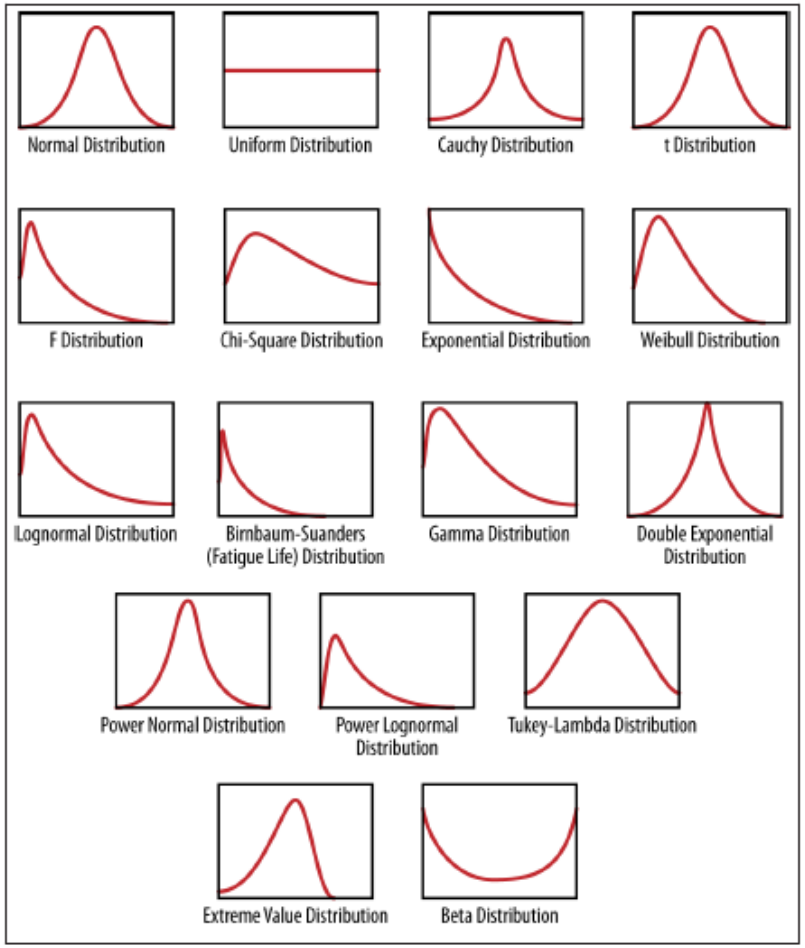
Unknown Parameter

$\omega \sim P(\Theta)$
?

Observations

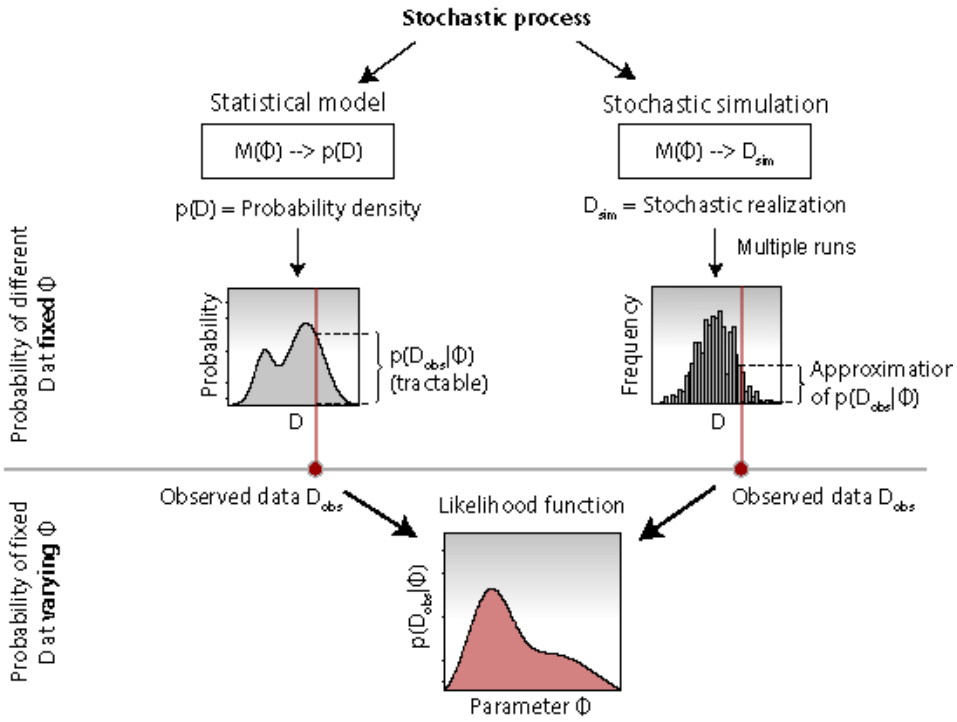
Probability Distributions

- A **probability distribution** is used to describe the potential outcome of a random variable. $\theta \sim P(\Theta)$?



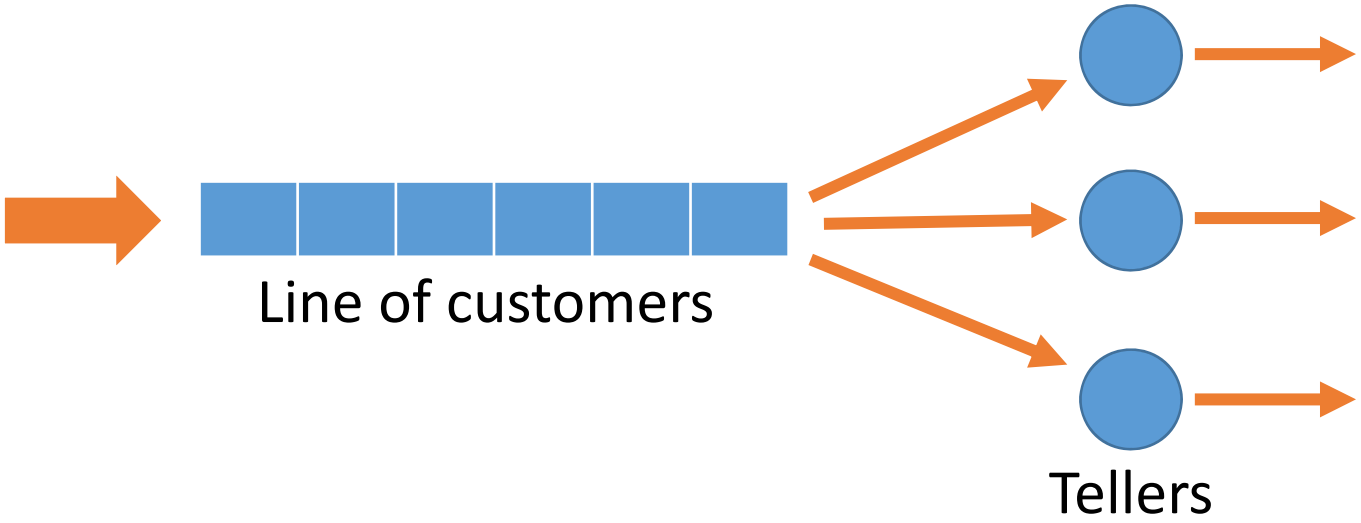
Stochastic simulation

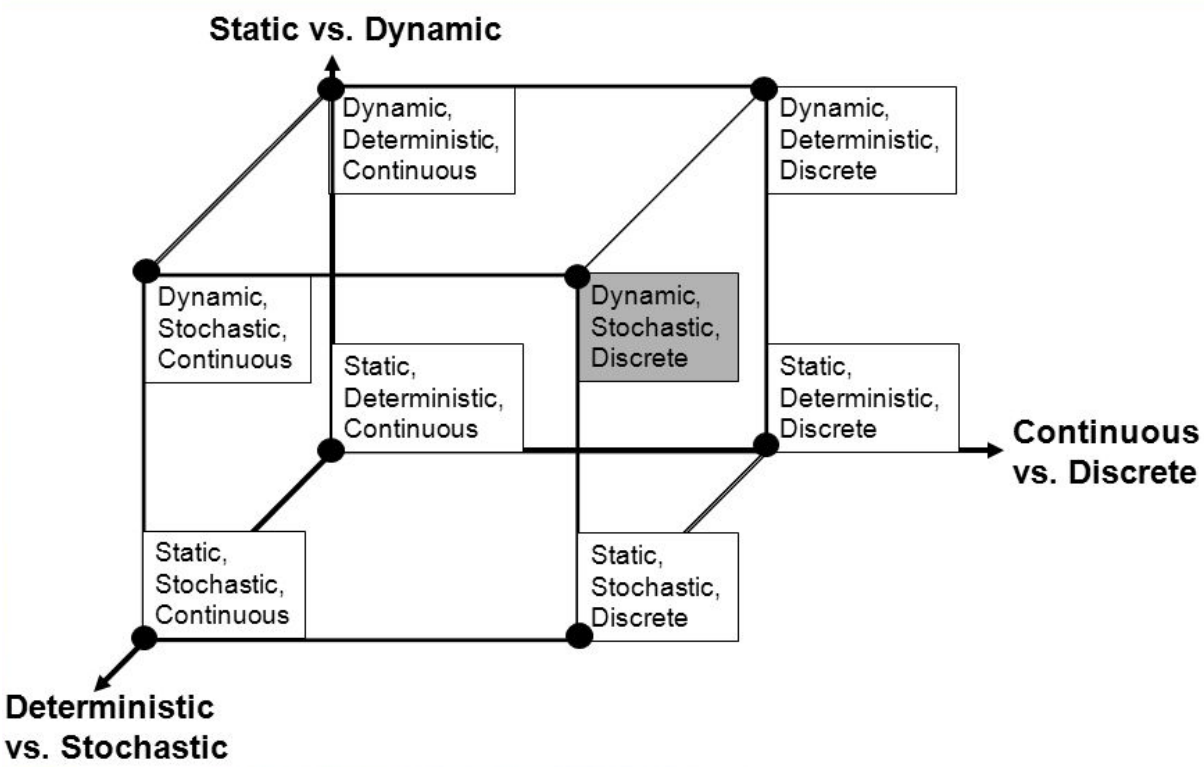
- A **stochastic simulation** is a simulation that traces the evolution of random variables that can change stochastically (randomly) with certain probabilities.



Deterministic vs. Stochastic

Queueing model for bank (customer & tellers)



Deterministic	Stochastic
• Customer arrives every 10 minutes during 9:00-10:59 and every 5 minutes during 11:00-12:00 • Each customer will take 12 minutes of Teller's time.	• Next customer will arrive 8-11 minutes, using uniform distribution. • The time a customer will take with the teller is normally distributed, with mean of 9 minutes and s.d. of 2.5 minutes.

Simulation Full Models