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Probability in Statistics

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Introduction

Probability can be defined as the chance of an event occurring .

Sample space and Probability

A probability experiment is a chance process that leads to well-defined results called outcomes.

An outcome is the result of trial of probability experiment .

A sample space is the set of all possible outcomes of probability experiment.

Introduction

An event consists of a set outcomes of probability experiment.

There are 3 types of probability

1. Classical probability
2. Relative frequency (Empirical probability)
3. Subjective probability

Equally likely events are events that have the same probability of occurring.

Classical Probability

- The probability of any event E is

$$P(E) = \frac{n(E)}{n(S)}$$

n(E) is the number of outcomes in E

n(s) is the total number of outcomes in the sample space.

<u>Example</u>

Find the probability of getting a black 10 when drawing a card from a deck. (1/26)

Classical Probability

Example

A card is drawn from an ordinary deck. Find the probability of

a. getting a jack

b. getting the 6 of clubs

c. getting a 3 or a diamond

d. getting a 3 or a 6

Relative frequency probability

- The relative frequency probability is

$$P = \frac{f}{n}$$

f is the number of event occurred

n is the total number of observations

Subjective probability

- Subjective probability is an estimate based on experience or intuition
- Which method ?

1. The chance that you will get married in next year is zero ....
2. Based on the government data ,the chance in an automobile is about 1 in 8,000 per year...................
3. The chance of rolling a 6 with a fair die........................

The probability rules

There are 4 basic probability rules. These rules are helpful in solving probability problems.

1. The probability of any event E is a number between and including 0 and 1 .

$$0 \leq p(E) \leq 1$$

2. If an event cannot occur ,its probability is 0

3. If an event is certain ,then the probability of E is 1

4. The sum of probabilities of all the outcomes in the sample space is 1

The probability rules

<u>The complement of an event E</u>

is the set of outcomes in the sample space that are not included in the outcomes of event E . The complement of E is denoted by $\bar{E}$ (E bar)

<u>Rule for complement events</u>

$$p(\overline{E}) = 1 - p(E)$$

The complementary events can be represented by *<u>Venn diagram</u>*

The addition rules for probability

Two events are mutually exclusive event if they cannot occur at the same time.

1. The person is a female
2. The person live in Chiang mai

<u>Two events are not mutually exclusive events</u>

1. The person is a female
2. The person is male

<u>Two events are mutually exclusive events</u>

The addition rules for probability

The probability of two or more events can be determined by the addition rules . The first addition rule is used when the events are mutually exclusive event.

<u>Addition rule 1</u>

When two events A and B are mutually exclusive events, the probability that A and B will occur is

$$P(A \text{ or } B) = p(A)+P(B)$$

The addition rules for probability

The probability of two or more events can be determined by the addition rules . The first addition rule is used when the events are mutually exclusive event.

Addition rule 2

If A and B are not mutually exclusive , then

$$P(A \text{ or } B) = p(A)+P(B) - P(A \text{ and} B)$$

Example 2

- Based on a random sample of 200 adults classified by gender and by highest education level attained

Gender /education	Elementary	Secondary	College	Total
Male	38	28	22	88
Female	45	50	17	112
Total	83	78	39	200

Expected value or Mean : E(X)

The expected value of a variable is the weighted average of all its possible values.

$$E(X) = x_1 p(X = x_1) + x_2 p(X = x_2) + \ldots\ldots + x_n p(X = x_n)$$

Example Find the mean of the number of heads that occur.

X	P(X=x)
0	1/8
1	3/8
2	3/8
3	1/8

Example

- The insurance company involve two distinct events:

1. In the event that a person buys a policy , the price of the policy is $250
2. In the event that a person is paid for claim ,the value for claim is $100,000 the probability for this event is 1/500. Find the expected value of each insurance policy .

Example

The probability distribution shown represents the number of trips of five nights or more that American adult take per year. That is 6% do not take any trip lasting five nights or more, 70% take one trip lasting five nights or more per year etc.) Find the mean .

Number of trips	**0**	**1**	**2**	**3**	**4**
P(X=x)	0.06	0.70	0.20	0.03	0.01

Example

Application

One thousand tickets are sold at \$1 each for a color television valued at \$350 . What is expected value of the gain if you purchase one ticket ?

Normal Distribution

- In this topic , we learn how researcher determine normal distribution interval for specific case.
- Normal random variable is continuous that can assume all values between any two value of the variable.

For example :

The heights of adult men

The lifetimes of battery e

etc.

Normal Distribution

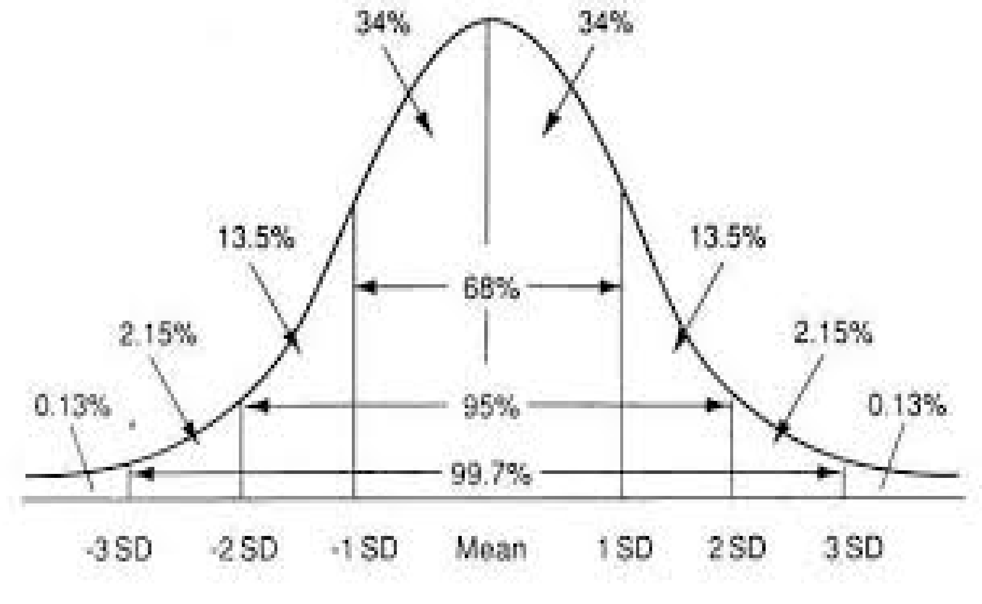
- The normal distribution is known as a *bell curve or a guassian* distribution, named for the German mathematician *Carl Friedrich Guass (1777-1855)*
- When the data values are evenly distributed about the mean , a distribution is said to be a systematic distribution
- When the majority of data values fall to the right of the mean , the distribution said to be skew to left hand side.
- When the majority of data values fall to the left of the mean , the distribution said to be skew to right hand side.

Normal Distribution

- A Normal distribution is a continuous distribution.
- The properties of the theoritical normal distribution

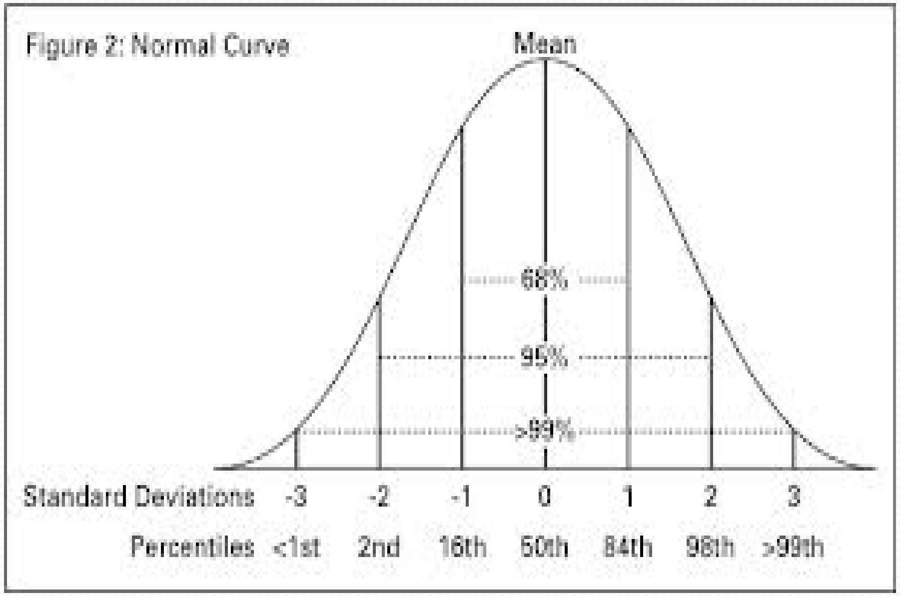
1. A normal distribution is bell-shaped.
2. The mean median and mode are equal located at the center of the distribution.
3. The total area under the normal distribution curve is equal to 1.00
4. The area under the part of a normal curve that is lies within 1 standard deviation is 0.68(68%) 2 standard deviation about 0.95(95%) and within 3 standard deviation about 0.997(99.7%)

Normal distribution



The Standard Normal Distribution

- The standard normal distribution is a normal with mean of 0 and a standard deviation is 1



- The values under the curve indicate the proportion of area in each section.

The Standard Normal Distribution

All normal distributed variables can be transformed into the standard normal distribution by using the standard score (z score) :

$$z = \frac{x - \mu}{\sigma}$$

If we know the value of z score , we can find the area under the standard normal distribution curve by using the standard normal curve table. (using the statistical table) .

The Standard Normal Distribution

Find the area under the curve (probability) of the standard normal distribution.

a. $P(z<0.45)= 0.6736$

b. $P(z>1.56)= 1-P(z<1.56)= 1-0.9406= 0.0594$

c. $P(z<-2.44)=1-P(z<2.44)= 1-0.9927=0.0073$

d. $P(z>-1.56)= P(z<1.56)= 0.9406$

e. $P(0.4<z<1.6)=P(z<1.6)-P(z<0.4)$

$= 0.9452-0.6554=0.2898$

The Standard Normal Distribution

Find the area under the curve (probability) of the standard normal distribution.

f. $P(-0.4<z<1.6)=p(Z<1.6)-P(z<-0.4)$

$= p(z<1.6)- \{1-P(Z<0.4)\}$

$= 0.9452-\{1-0.6554\}=0.6006$

g. $P(-1.35<z<-0.56)=p(z<1.35)-p(z<0.56)$

$= 0.9115-0.7123=0.1992$

h. $P(0<z<2.32) = p(z<2.32)-p(z<0)$

$= 0.9898-0.5 = 0.4898$

The Standard Normal Distribution

Application of the Normal Distribution

A survey found that women spend , on average \$146.21 on beauty products during the summer months. Assume the standard deviation is \$29.44 . Find the percentage of women who spend less than \$160.00 . Assume the variable is normal distributed.

Solution Let X be the money that women spend during summer that is normal distribution

we want to find $P(X<160)$

we have $\mu=46.21$ and $\sigma=29.44$

so that

The Standard Normal Distribution

we want to find P(X<160)

we have μ=46.21 and σ=29.44

so that

$$p(X < 160) = p(Z < \frac{160 - 146.21}{29.44})$$

$$p(X < 160) = p(Z < 0.47) = 0.6808$$

Therefore ,0.6808=68.08% ,of women spend less than $160 on beauty products during the summer month.

The Standard Normal Distribution

Application of the Normal Distribution

Each month, an American household generates an average of 28 pounds of newspaper for garbage or recycling. Assume the standard deviation is 2 pounds. If a household is selected at random, find the probability of its generating

a. Between 27 and 31 pounds per month

b. More than 30.2 pounds per month

Assume the variable is approximately normally distributed.

The Standard Normal Distribution

Application of the Normal Distribution

Solution Let X be the household garbage.

We have $\mu=28$ and $\sigma=2$

a. $P(27<X<31) = p(\frac{27-28}{2} < Z < \frac{31-28}{2})$

$= p(-0.5 < Z < 1.5)$

$= 0.6247 = 62.47\%$

The probability that a randomly selected household generates between 27 and 31 of newspaper per month is 62.47%.

The Standard Normal Distribution

Application of the Normal Distribution

Solution Let X be the household garbage.

We have $\mu=28$ and $\sigma=2$

b. $P(X>30.2) = p(Z > \frac{30.2-28}{2})$

$= p(Z > 1.1)$

$= 1 - P(Z<1.1) = 1 - 0.8643 = 0.1357$

The probability that a randomly selected household will accumulate more than 30.2 pounds of newspapers is 0.1357= 13.57%.

The Standard Normal Distribution

Application of the Normal Distribution

Thai people consume an average of 1.64 cup of coffee per day. Assume the variable is approximately normally distributed with a standard deviation of 0.24 cup. If 500 individuals are selected, approximately how many will drink less than 1 cup of coffee per day ?

(0.0038, 2 people)

solution

Let X be the number of cup of coffee that Thai people consume per day.

The Standard Normal Distribution

solution

Let X be the number of cup of coffee that Thai people consume per day

We have μ=1.64 and σ=0.24

We want to find P(X<1)

$$p(x<1) = p(Z < \frac{1-1.64}{0.24}) \quad = p(Z < -2.67)$$

= 1-p(z<2.67)= 1-.9962 = 0.0038

To find how many people drank less than 1 cup of coffee multiply the sample size 500 by 0.0038 = 1.9 round up to 2 people . Hence approximately 2 people will drink less than 1 cup of coffee a day.