

## Problem Set 6 — Due 10 September

Turn in all problems that you have correctly solved. If you work with someone on a problem, turn in a single writeup for it.

**Problem 13:  $\text{HC} \propto \text{HP}$**  For the midterm you reduced HAMILTONIAN PATH (HP) to HAMILTONIAN CIRCUIT (HC). Now give a (direct) reduction for the other direction: show how to reduce HC to HP.

**Problem 14: HALF-CLIQUE** Consider the following problem:

HALF CLIQUE:

**Instance:** A graph  $G = (V, E)$

**Question:** Does  $G$  contain a clique of size  $n/2$ , where  $n = |V|$ ?

Show that HALF CLIQUE is NP-Complete. In other words, show that CLIQUE remains hard when restricted to  $K = |V|/2$ .

**Problem 15: CLUSTERING** Consider the following problem:

CLUSTERING:

**Instance:** A finite set  $X$  and a distance (a nonnegative number)  $d(x, y)$  between each  $x, y \in X$ ; also numbers  $K$  (the numbers of clusters) and  $B$  (the compactness of each cluster).

**Question:** Can  $X$  be partitioned into disjoint sets  $X_1, \dots, X_K$  such that for any  $i \in [1..K]$  and any  $x, y \in X_i$ ,  $d(x, y) \leq B$ ?

Prove that CLUSTERING is NP-Complete. *Hint:* What other problem have we discussed that speaks of partitioning vertices into disjoint subsets?